Instantaneous center of rotation and singularities of planar parallel manipulators

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Abstract With regard to planar parallel manipulators, a general classification of singularities into three groups is given. The classification scheme relies on the properties of instantaneous centers of rotation. This method is very fast and can easily be applied to the manipulators under study. The method is applied to a planar three-degrees-of-freedom parallel manipulator and all its singular configurations are found.

Keywords planar; parallel manipulator; singularity; instantaneous center of rotation

Introduction

Parallel manipulators consist of multiple branches acting on a common payload platform. They have superiorities over serial ones, which include greater stiffness, improved accuracy and dynamic characteristics, higher payload/weight ratio and higher operating speeds. These advantages stem from multiple support and the fact that all the motors are fixed to the base. However, near singular configurations, all manipulators experience poor performances, and parallel manipulators are not exempt form this rule. Therefore, they lose their advantages over serial manipulators at these configurations.

A manipulator singularity occurs at the coincidence of different direct or inverse kinematic solutions. The latter are understood here as the computation of the values of driving-joint variables from given Cartesian variables, while the former (direct kinematics) are defined as the computation of the values of the Cartesian variables from given driving-joint variables. Algebraically, singularity amounts to a rank deficiency of the Jacobian matrices; geometrically, singularity is observed whenever the manipulator gains some additional, uncontrollable degrees of freedom (dof), or loses some dof. Similarly, the force transmission performance of a parallel manipulator is very poor near singular configurations.

The concept of singularity has been extensively studied in connection with serial manipulators [1–3]. As regards manipulators with kinematic loops, the literature is more limited [4–10]. Litvin *et al*. [2] used coordinate transformation matrices to locate singular configurations, while Merlet [5] proposed a method based on Grassmann line geometry. Zlatanov *et al*. [6, 7] classified singularities via motion–space and velocity–space models. Notash [8] found singular configurations based on the concept of screw theory. However, the classical method to locate singular configurations relies on the properties of the Jacobian matrices of the manipulator [9].

All these methods have the same drawback: they rely on cumbersome mathematical or geometrical computations. Here, we introduce a fast method of finding singular configurations of planar parallel manipulators, based on the properties of *instantaneous centers of rotation*. Moreover, the method is easy to implement.

Instantaneous center of rotation

The instantaneous center of rotation (ICR) is defined as the instantaneous location of a pair of coincident points of two rigid bodies, the absolute velocities of which are equal. In the other words, one rigid body can rotate about the ICR, relative to the other one. The ICR is defined between any two rigid bodies that have relative planar motion. Therefore, there are three ICRs between three rigid bodies in relative motion. They are related by the following well known theorem.

Theorem 1: the Arnhold–Kennedy theorem

The three instant centers of rotation shared by three rigid bodies in relative motion to one another all lie on the same line.

There are three types of planar motion between two rigid bodies, namely, translation, rotation and general motion. In translation, the relative velocity of all points are parallel, thus, the ICR lies on a line perpendicular to the velocity vector at infinity. In rotation, the ICR is trivial, but any relative general motion can be regarded as a pure rotation about the ICR. Therefore, as a direct consequence of the Arnhold–Kennedy theorem, one can propose theorem 2.

Theorem 2

There is no relative motion between two rigid bodies in the absence of any ICR between them.

Singularity analysis

In serial manipulators, singularities occur whenever one actuator does not produce any motion of the end effector (EE). In parallel manipulators, in addition to the foregoing, we have another type of singularity, in which the EE cannot resist forces or torques in one or more directions, even if all the actuators are locked. Thus, we classify singularities into three groups:

(1) The first type of singularity occurs when one actuator does not produce any motion of the EE. This type of singularity consists of a point or a set of points where different branches of the inverse kinematic problem meet. It is supposed that each motor moves the EE in one direction. Therefore, considering the motion of only one actuator, the EE is fixed and the Cartesian velocity vector related to the motion of that actuator cannot be produced. In this case one can find all ICRs while the EE is fixed. Thus, all the rigid bodies, including the actuator under study, move except the EE, i.e., that actuator cannot move the EE.

(2) The second type of singularity occurring only in parallel manipulators consists of a point or a set of points where different branches of the direct kinematic problem meet. This can easily be proven whenever one can find the ICR of the EE with respect to the base, even if the actuators are locked. In this case the EE can rotate about the ICR relative to the base. Thus, the EE can move in one or more directions and cannot resist a force or a torque in those directions, while the actuators are locked. Therefore, the EE gains some additional uncontrollable dof.

(3) The third type of singularity occurs whenever we have both types of the above singularities simultaneously. Under a singularity of this type, configurations arise in which the EE can undergo finite motion even if the actuators are locked, or, equivalently, it cannot resist forces or torques in one or more directions even if the actuators are locked. Further, a finite motion of the actuators produces no motion of the EE and some of the Cartesian velocity vectors cannot be produced.

Planar three-dof parallel manipulators

In this section the three types of singularities discussed above are investigated for a planar three-dof parallel manipulator as depicted in Fig. 1. The manipulator consists of a platform supported by three legs, each of which has one revolute joint. The dof of the manipulator is determined by means of the Chebyshev–Grubler–Kutzbach formula [11] for planar manipulators, as follows:

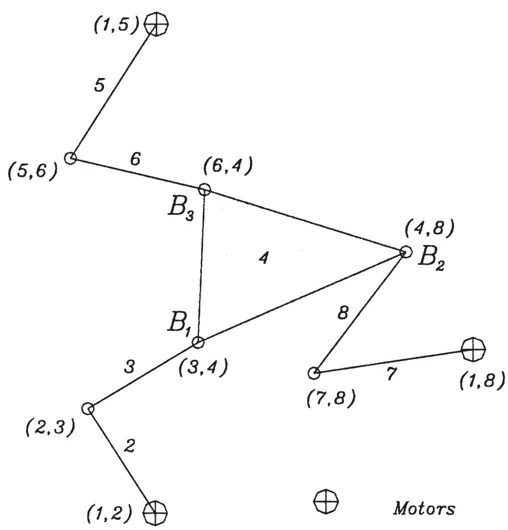

Fig. 1 *Planar three-dof parallel manipulator.*

$$f = 3(n-1-k) + \sum_{i=1}^{k} f_i$$

in which n and f_i are the number of links and the dof of the ith joint, respectively, and k is the number of joints. Therefore, the dof of the manipulator under study is:

$$f = 3(8-1-9) + 9 = 3$$

Thus, the three fixed motors move the EE in the plane.

The first type of singularity

This type of singularity occurs whenever one of the actuator does not move the EE. Considering one of the legs of the manipulator as depicted in Fig. 2, one can find ICRs as follows. The ICR between links one and two is shown as (1,2). Point (2,3) is the ICR between links two and three, while point (3,4) is the ICR between links three and four, which is the EE of the manipulator. According to theorem 1, the ICR between links two and four, namely point (2,4), lies on the line joining (2,3) and (3,4). Similarly, the ICR between links one and three lies on the line joining (1,2) and (2,3). If this leg is fully extended or folded, the EE remains fixed and ICRs (1,3) and (3,4) coincide, while the motor actuating link two is moving. Therefore, link two rotates about ICR (1,2) relative to the base, while link three rotates about ICR (1,3) relative to the fixed EE. In this case the corresponding motor cannot move the EE, thus the EE cannot move in the direction of the extended or the folded legs and the manipulator loses one degree of freedom (Figs 3 and 4).

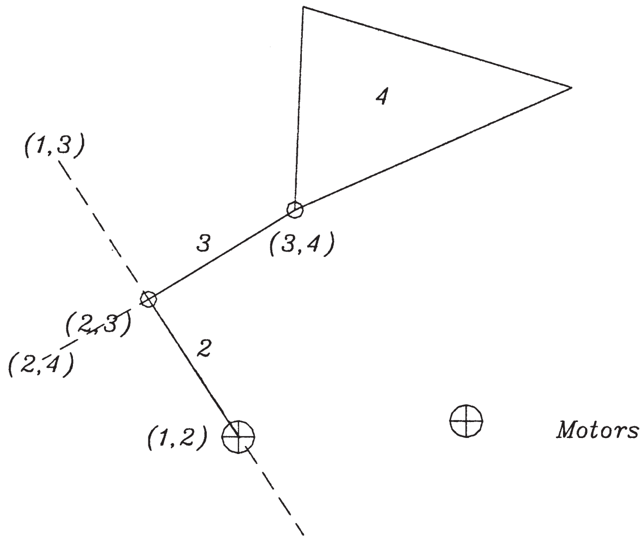

Fig. 2 *One leg of the manipulator.*

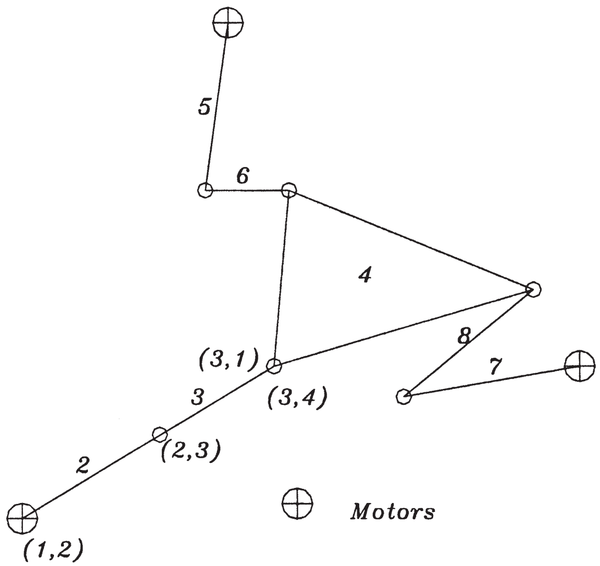

Fig. 3 *Example of the first type of singularity in which one leg is fully extended.*

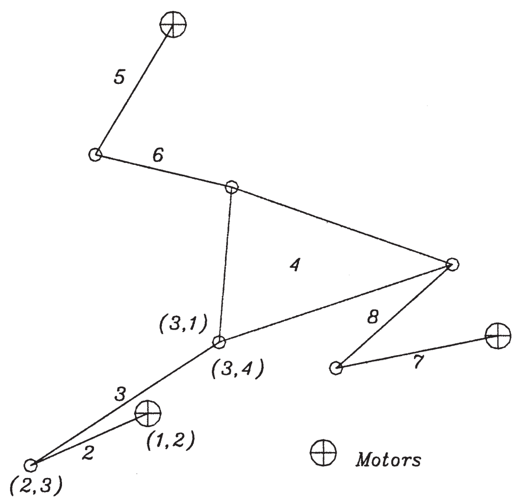

Fig. 4 *Example of the first type of singularity in which one leg is folded.*

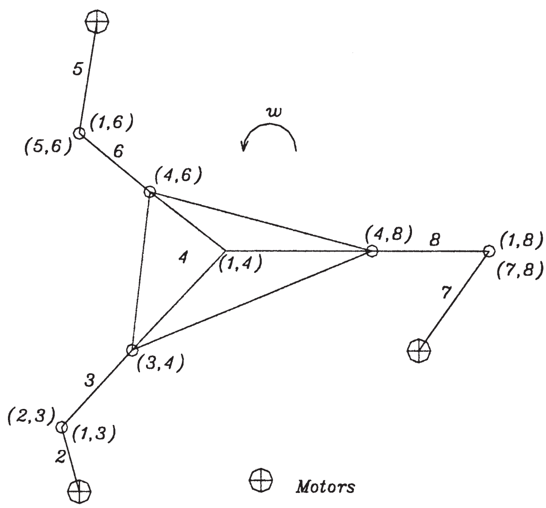

Fig. 5 *Example of the second type of singularity in which links three, six and eight intersect at a point.*

The second type of singularity

This type of singularity occurs whenever the EE can move relative to the base, even if all the actuators are locked. If the motors are locked, links two, five and seven are fixed to the base and ICRs (1,3), (1,6) and (1,8) are equivalent to those of (2,3), (5,6) and (7,8), respectively. According to theorem 1, ICR (1,4) lies on the line joining (1,3) and (3,4). Moreover, it is located on the line joining (1,6) and (4,6). Therefore, ICR (1,4) is the intersection of these two lines. Similarly, ICR (1,4) is the intersection of lines joining (1,3) to (3,4) and (1,8) to (4,8). We have this type of singularity in the case where two points coincide. If this is the case, the EE can rotate about that point, while the actuators are locked.

Two different cases for which we have this type of singularity can be identified. The first occurs when three lines bearing links three, six and eight intersect at a common point. This point is the ICR of link four with respect to the base, which means the EE can rotate freely about that point relative to the base, even if the actuators are locked. Thus, the manipulator gains one rotational dof. Moreover, any torque applied to the EE cannot be balanced by the actuators (Fig. 5).

The second case in which we have this type of singularity occurs whenever the three lines bearing links three, six and eight are parallel. Therefore, they intersect at infinity. Thus, the EE can rotate about that point relative to the base, i.e., the EE can translate along **u** even if the actuators are locked, and the manipulator gains one translational degree of freedom along **u**. Moreover, the EE cannot balance forces along this direction (Fig. 6).

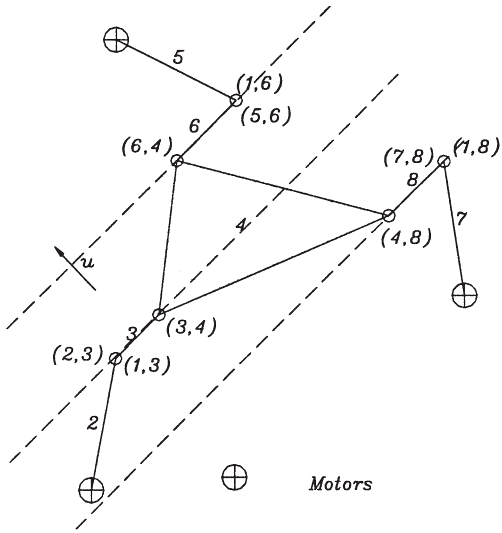

Fig. 6 *Example of the second type of singularity in which links three, six and eight are parallel.*

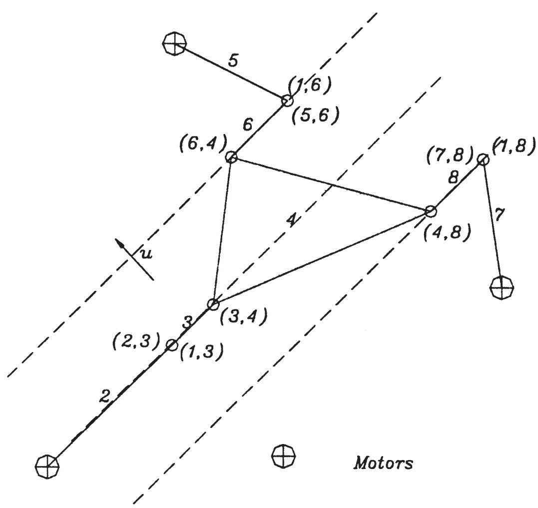

Fig. 7 *Example of the third type of singularity.*

The third type of singularity

This type of singularity occurs whenever both types of the foregoing singularities occur simultaneously. We have this type of singularity whenever the three lines bearing links three, six and eight are either parallel or concurrent at a common point and at least one leg is fully extended or folded (Figs 7 and 8). At these configura-

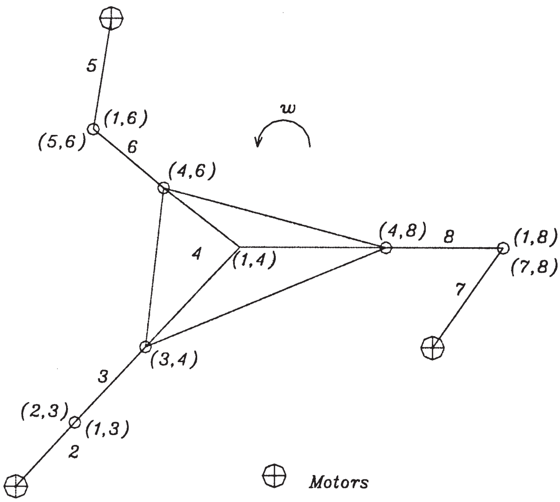

Fig. 8 *Example of the third type of singularity.*

tions the motion of at least one actuator does not produce any motion of the EE. Therefore, the manipulator loses one or more dof. Further, the EE can move freely in one or more directions even if all the actuators are locked and some forces or torque applied to it cannot be balanced by the actuators. Thus, the manipulator gains one or more uncontrollable dof.

Conclusions

A classification of the singularities of parallel manipulators into three groups has been given. The three types of singularities, which have different kinematic interpretations, were identified for a planar parallel manipulator. The simplicity and the robustness of the method make it possible to implement it for all planar parallel manipulators.

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