

---

# Thermophysical properties of ice/water/steam from $-20^{\circ}\text{C}$ to $50^{\circ}\text{C}$

P. E. Liley

3608 Mulberry Drive, Lafayette, IN, 47905-3937, USA

E-mail: eandpliley@insightbb.com

**Abstract** The properties of water substance (vapor pressure, enthalpy, specific heat capacities and their ratio, viscosity, thermal conductivity, Prandtl number, speed of sound, thermal expansivity and surface tension) are tabulated at close temperature increments from  $-20^{\circ}\text{C}$  to  $50^{\circ}\text{C}$ . Simple equations which accurately fit these values are given.

**Keywords** ice/water/steam properties

## Nomenclature

- $c$  specific heat capacity,  $\text{kJ}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$   
 $h$  specific enthalpy,  $\text{kJ}\cdot\text{kg}^{-1}$   
 $k$  thermal conductivity,  $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$   
 $P$  pressure, kPa  
Pr Prandtl number,  $\mu c_p/k$   
 $t$  temperature,  $^{\circ}\text{C}$   
 $v_s$  speed of sound,  $\text{m}\cdot\text{s}^{-1}$   
 $\beta$  isobaric thermal expansivity,  $1/v(\partial v/\partial t)_p$ ,  $\text{K}^{-1}$   
 $\gamma$  ratio of principal specific heat capacities ( $c_p/c_v$ )  
 $\mu$  viscosity,  $\mu\text{Pa}\cdot\text{s}$   
 $\tau$  surface tension,  $\text{mN}\cdot\text{m}^{-1}$

## Subscripts

- $c$  condensed phase  
 $f$  liquid  
 $g$  gas  
 $p$  at constant pressure  
 $s$  solid  
 $v$  at constant volume

For many years the recommended source of thermodynamic property values for water substance has been the tables of Haar *et al.* [1]. However, some omissions in coverage occur. There are no tables of the specific heat capacities of the saturated liquid and vapor, tables of transport properties for these states are given only at intervals of 10 K and no information on the solid state (ice) appears. For such properties the tables of Grigull *et al.* [2] are far preferable. A more comprehensive set only for the usual range  $0$ – $100^{\circ}\text{C}$  was given by Aleksandrov and Trakhtenhertz in a publication of rather limited distribution [3]. An earlier set, from  $0^{\circ}\text{C}$  to  $150^{\circ}\text{C}$  and to

TABLE 1 *Thermophysical properties of ice/water/steam from  $-20^{\circ}\text{C}$  to  $50^{\circ}\text{C}$* 

T ( $^{\circ}\text{C}$ )	P (kPa)	$h_c$ (kJ/kg)	$h_{cg}$ (kJ/kg)	$h_g$ (kJ/kg)	$c_{pc}$ (kJ/kg-K)	$c_{pg}$ (kJ/kg-K)	$c_{vc}$ (kJ/kg-K)	$c_{vg}$ (kJ/kg-K)	$\gamma_l$	$\gamma_g$
-20	0.1039	-374.0	2837.8	2463.8	1.929	1.8683		1.4047		1.3300
-15	0.1160	-364.2	2837.1	2473.2	1.964	1.8711		1.4073		1.3296
-10	0.2606	-354.1	2836.5	2482.4	1.998	1.8752		1.4108		1.3292
-5	0.4023	-343.8	2835.5	2491.7	2.033	1.8796		1.4145		1.3288
0	0.6112	-333.4	2834.3	2500.9	2.067	1.8843		1.4184		1.3285
0	0.6112	0.000	2500.94	2500.94	4.2199	1.8843	4.2174	1.4184	1.0006	1.3285
2	0.7057	8.390	2496.18	2504.57	4.2136	1.8864	4.2128	1.4201	1.0002	1.3284
4	0.8136	16.812	2491.41	2508.22	4.2080	1.8884	4.2079	1.4217	1.0000	1.3283
6	0.9354	25.223	2486.66	2511.88	4.2033	1.8905	4.2026	1.4234	1.0002	1.3281
8	1.0730	33.625	2481.92	2515.55	4.1992	1.8926	4.1970	1.4252	1.0005	1.3280
10	1.2282	42.020	2477.17	2519.19	4.1956	1.8947	4.1910	1.4269	1.0010	1.3278
12	1.4030	50.409	2472.44	2522.91	4.1924	1.8969	4.1847	1.4286	1.0017	1.3278
14	1.5989	58.791	2467.69	2526.48	4.1900	1.8991	4.1782	1.4304	1.0025	1.3276
16	1.8187	67.169	2462.97	2530.14	4.1878	1.9013	4.1713	1.4322	1.0036	1.3275
18	2.0646	75.543	2458.26	2533.80	4.1859	1.9036	4.1643	1.4340	1.0050	1.3274
20	2.3392	83.913	2453.54	2537.45	4.1844	1.9059	4.1570	1.4359	1.0065	1.3273
22	2.6452	92.283	2448.79	2541.09	4.1830	1.9082	4.1495	1.4377	1.0081	1.3273
24	2.9858	100.653	2444.06	2544.71	4.1821	1.9106	4.1418	1.4396	1.0095	1.3272
26	3.3638	109.015	2439.56	2548.34	4.1812	1.9130	4.1339	1.4414	1.0114	1.3272
28	3.7830	117.373	2434.56	2551.93	4.1806	1.9155	4.1258	1.4433	1.0130	1.3272
30	4.2470	125.734	2429.82	2555.55	4.1801	1.9180	4.1176	1.4453	1.0152	1.3271
32	4.7596	134.094	2425.06	2559.15	4.1798	1.9206	4.1093	1.4472	1.0173	1.3271
34	5.3251	142.454	2420.30	2562.75	4.1797	1.9232	4.1001	1.4492	1.0193	1.3271
36	5.9479	150.814	2415.54	2566.34	4.1795	1.9259	4.0916	1.4512	1.0214	1.3272
38	6.6328	159.173	2410.76	2569.93	4.1796	1.9286	4.0828	1.4532	1.0236	1.3272
40	7.3848	167.533	2405.98	2573.51	4.1796	1.9314	4.0737	1.4553	1.0260	1.3272
42	8.2093	175.894	2401.19	2577.08	4.1798	1.9343	4.0644	1.4574	1.0283	1.3273
44	9.1121	184.255	2396.39	2580.65	4.1802	1.9373	4.0551	1.4595	1.0308	1.3274
46	10.0992	192.616	2391.58	2584.20	4.1805	1.9404	4.0456	1.4617	1.0333	1.3275
48	11.1769	200.979	2386.77	2587.75	4.1810	1.9435	4.0361	1.4640	1.0359	1.3276
50	12.3521	209.343	2381.94	2591.28	4.1815	1.9468	4.0264	1.4663	1.0385	1.3277
Basis	[5]	[6]	[6]	[6]	[6, 10]	[6]	[6]	[ $\sigma$ ]	<sup>a</sup>	<sup>a</sup>

At  $0^{\circ}\text{C}$  the liquid (second entry for  $0^{\circ}\text{C}$ ) is subcooled by  $0.01^{\circ}$  below the triple point; the first entry for  $0^{\circ}\text{C}$  is for the condensed phase.

<sup>a</sup>Calculated from  $c_p$  and  $c_v$  values.

<sup>b</sup>Calculated from  $c_p$ ,  $\mu$  and  $k$  values.

<sup>c</sup>Calculated from volume values [5].

$\mu_r$ ( $\mu\text{Pa}\cdot\text{s}$ )	$\mu_g$ ( $\mu\text{Pa}\cdot\text{s}$ )	$k_c$ (W/m·K)	$k_g$ (W/m·K)	$\text{Pr}_r$	$\text{Pr}_g$	$v_{sf}$ (m/s)	$v_{sg}$ (m/s)	$\beta_c$ (k/K)	$\beta_g$ (k/K)	$\tau$ (mN/m)	$t$ (°C)
	8.75	2.420	0.01604		1.0192		394.29	147.5			-20
	8.88	2.369	0.01630		1.0193		398.00	151.0			-15
	8.99	2.318	0.01654		1.0192		401.69	154.5			-10
	9.11	2.267	0.01680		1.0190		405.36	158.0			-5
	9.22	2.216	0.01707		1.0187			161.5			0
1792	9.22	0.5610	0.01707	13.45	1.0187	1402.3	409.00	-0.681	3.692	75.66	0
1674	9.27	0.5648	0.01718	12.49	1.0186	1412.0	410.44	-0.328	3.667	75.37	2
1568	9.32	0.5686	0.01728	11.60	1.0185	1421.4	411.88	0.002	3.644	75.08	4
1472	9.37	0.5725	0.01739	10.81	1.0183	1430.4	413.31	0.312	3.620	74.80	6
1385	9.42	0.5763	0.01751	10.09	1.0182	1438.9	414.74	0.604	3.597	74.51	8
1306	9.47	0.5800	0.01762	9.447	1.0180	1447.0	416.17	0.879	3.573	74.22	10
1234	9.52	0.5837	0.01774	8.863	1.0178	1454.8	417.58	1.141	3.550	73.93	12
1168	9.57	0.5874	0.01786	8.331	1.0176	1462.1	418.99	1.389	3.527	73.65	14
1108	9.62	0.5911	0.01798	7.850	1.0174	1469.1	420.40	1.625	3.505	73.34	16
1053	9.68	0.5948	0.01810	7.410	1.0174	1475.8	421.79	1.851	3.484	73.04	18
1002	9.73	0.5984	0.01823	7.007	1.0172	1482.2	423.18	2.067	3.464	72.74	20
954.8	9.79	0.6020	0.01836	6.634	1.0170	1488.1	424.56	2.275	3.444	72.42	22
911.0	9.84	0.6055	0.01849	6.292	1.0168	1493.8	425.94	2.474	3.425	72.11	24
870.4	9.90	0.6089	0.01862	5.977	1.0167	1499.1	427.31	2.666	3.407	71.81	26
832.6	9.95	0.6122	0.01875	5.686	1.0165	1504.2	428.67	2.852	3.389	71.51	28
797.4	10.01	0.6155	0.01889	5.415	1.0164	1508.9	430.03	3.032	3.372	71.19	30
764.6	10.07	0.6187	0.01903	5.165	1.0163	1513.4	431.38	3.206	3.356	70.87	32
733.9	10.13	0.6218	0.01917	4.933	1.0162	1517.7	432.72	3.374	3.339	70.56	34
705.1	10.19	0.6248	0.01931	4.717	1.0160	1521.6	434.06	3.538	3.322	70.24	36
678.2	10.25	0.6278	0.01945	4.515	1.0158	1525.3	435.39	3.698	3.305	69.92	38
653.0	10.31	0.6306	0.01960	4.329	1.0157	1528.7	436.71	3.853	3.289	69.60	40
629.2	10.37	0.6334	0.01975	4.152	1.0156	1531.9	438.02	4.005	3.273	69.27	42
606.8	10.43	0.6361	0.01990	3.988	1.0155	1534.9	439.33	4.153	3.258	68.94	44
585.6	10.50	0.6387	0.02006	3.833	1.0154	1537.6	440.63	4.298	3.244	68.51	46
565.7	10.56	0.6412	0.02021	3.689	1.0153	1540.1	441.92	4.438	3.230	68.28	48
546.8	10.62	0.6436	0.02036	3.553	1.0152	1542.4	443.21	4.578	3.217	67.94	50
[9]	[9]	[8]	[8]	<sup>b</sup>	<sup>b</sup>	[6]	[6]	[3, 4]	<sup>c</sup>	[11]	

TABLE 2 Equations for thermophysical properties

Property	Units	Range	Eq.	a	b	c	$\Delta'$
s.v.p.	kPa	A	L	22.01251	-6166.44	274.005	0.002
s.v.p.		B	L	16.99268	-4206.29	240.566	0.002
s.v.p.		C	L	16.66775	-4306.02	235.152	0.001
$h_s$	kJ/kg	A	P	-333.40	2.11	$4 \times 10^{-3}$	0.017
$h_f$		B	P	4.20763	4.20763	$5.72917 \times 10^{-4}$	0.039
$h_f$		C	P	0.380	4.17716	$4.1667 \times 10^{-5}$	0.002
$h_{sg}$		A	P	2834.3	-0.265	$4.5 \times 10^{-3}$	0.006
$h_{fg}$		B	P	2500.93	-2.3609	$4.08 \times 10^{-4}$	0.003
$h_{fg}$		C	P	2500.93	-2.3609	$4.08 \times 10^{-4}$	0.004
$h_g$		A	P	2500.9	1.845	$5 \times 10^{-4}$	0.00
$h_g$		B	P	2500.8	1.8478	$-7.625 \times 10^{-4}$	0.001
$h_g$		C	P	2500.8	1.8478	$-7.625 \times 10^{-4}$	0.001
$c_{ps}$		A	P	2.067	$6.89 \times 10^{-3}$		0.00
$c_{pf}$		B	P	4.2200	$-3.02083 \times 10^{-3}$	$6.0069 \times 10^{-5}$	0.015
$c_{pf}$		C	P	4.1967	$-9.11067 \times 10^{-4}$	$1.21528 \times 10^{-5}$	0.026
$c_{pg}$		A	P	1.8844	$-1.035 \times 10^{-3}$	$1.15 \times 10^{-5}$	0.007
$c_{pg}$		B	P	1.8844	$9.91667 \times 10^{-4}$	$4.1667 \times 10^{-6}$	0.004
$c_{pg}$		C	P	1.8881	$7.2167 \times 10^{-4}$	$9.02778 \times 10^{-6}$	0.005
$c_{vf}$		B	P	4.2177	$-2.55417 \times 10^{-3}$	$-2.53472 \times 10^{-5}$	0.016
$c_{vf}$		C	P	4.2263	$-3.0825 \times 10^{-3}$	$-1.8403 \times 10^{-5}$	0.003
$c_{vg}$		A	P	1.4183	$8.350 \times 10^{-4}$	$7.50 \times 10^{-6}$	0.010
$c_{vg}$		B	P	1.4183	$8.29167 \times 10^{-4}$	$2.4306 \times 10^{-6}$	0.003
$c_{vg}$		C	P	1.42029	$6.94447 \times 10^{-4}$	$4.51389 \times 10^{-6}$	0.004
$\gamma_f$		B	P	1.0006	$-1.70833 \times 10^{-4}$	$2.25694 \times 10^{-5}$	0.019
$\gamma_f$		C	P	0.99423	$4.16667 \times 10^{-4}$	$9.375 \times 10^{-6}$	0.007
$\gamma_g$		A	P	1.3285	$-7.25 \times 10^{-5}$		0.008
$\gamma_g$		B	P	1.3285	$-8 \times 10^{-5}$	$1.0 \times 10^{-6}$	0.006
$\gamma_g$		C	P	1.3271	$1.20633 \times 10^{-5}$	$3.17461 \times 10^{-7}$	0.022
$\mu_f$	Pa.s	B	L	3.84288	384.838	105.503	0.012
$\mu_f$		C	L	3.37247	514.060	125.351	0.012

$\mu_g$	A	P	9.22	0.0235		0.032
$\mu_g$	B	P	9.22	0.02583		0.044
$\mu_g$	C	P	9.2102	0.024722		0.019
$k_s$	A	P	2.216	-0.0102	$6.9444 \times 10^{-5}$	0.00
$k_f$	B	P	0.5610	$1.92917 \times 10^{-3}$	$3.125 \times 10^{-6}$	0.007
$k_f$	C	P	0.6089	$2.26387 \times 10^{-3}$	$-1.07639 \times 10^{-5}$	0.005
$k_g$	A	P	0.01707	$5.4 \times 10^{-5}$	$2 \times 10^{-7}$	0.108
$k_g$	B	P	0.01707	$5.25 \times 10^{-5}$	$2.7778 \times 10^{-7}$	0.013
$k_g$	C	P	0.01710	$5.13889 \times 10^{-5}$	$2.7778 \times 10^{-6}$	0.011
$Pr_f$	B	L	-1.65537	469.685	110.401	0.013
$Pr_f$	C	L	-1.78023	501.834	114.643	0.010
$Pr_g$	B	P	1.01873	-7.91667 $\times 10^{-3}$		0.40
$Pr_g$	C	P	1.01670	-8.75 $\times 10^{-3}$		0.40
$\nu_{sf}$	B	P	1402.3	4.9375	$-4.6875 \times 10^{-2}$	0.007
$\nu_{sf}$	C	P	1411.11	4.2056	$-3.1597 \times 10^{-2}$	0.004
$\nu_{sg}$	A	P	409.0	0.7265	$-4.5 \times 10^{-4}$	0.013
$\nu_{sg}$	B	P	409.0	0.72417	$-7.639 \times 10^{-4}$	0.001
$\nu_{sg}$	C	P	408.91	0.73107	$-9.028 \times 10^{-4}$	0.002
$\beta_s$	A	P	161.5	0.70		0.00
$\beta_f$	B	P	-0.681	0.18096	$-2.7813 \times 10^{-3}$ *	0.150
$\beta_f$	C	P	-0.39630	0.14561	$-1.2310 \times 10^{-3}$ **	0.013
$\beta_g$	B	P	3.692	$-1.2542 \times 10^{-2}$	$5.9028 \times 10^{-5}$	0.020
$\beta_g$	C	P	3.67603	$-1.16111 \times 10^{-2}$	$4.8611 \times 10^{-5}$	0.022
$\tau$	B	P	75.66	-0.14042	$-3.125 \times 10^{-4}$	
$\tau$	C	P	75.66	-0.141	$-2.7778 \times 10^{-4}$	

\*Also add term  $dr^3$ ,  $d = 2.9945 \times 10^{-5}$ . Range is 5–25 °C.

\*\*Also add term  $dr^3$ ,  $d = 6.16667 \times 10^{-6}$ .

Range is Temperature range: A, -20–0 °C; B, 0–25 °C; C, 25–50 °C.

Eq.: Equation used for property ( $\psi$  = any property,  $t$  = °C)

L (logarithmic)  $\ln \psi = a + b/(t + c)$

P (polynomial)  $\psi = a + bt + ct^2$

$\Delta'$ : Average deviation ( $\psi_{calc.} - \psi_{table.}/\psi_{table.}$  over range listed (%).

1000 bar, also appeared [4]. More recently, newer formulations of the properties of water substance have appeared, one being the 1995 International Association for the Properties of Water and Steam (IAPWS) formulation (here abbreviated as IAPWS 95). A 75-page set of values of density, specific volume, specific enthalpy and specific entropy, along with an 18-page IAPWS release, was given by Harvey [5] for pressures to 1000 MPa and temperatures to 2000 °C. More recently, a detailed explanation of the calculation methods and tables based on this formulation have appeared [6], for integral temperatures in kelvin. The American Society of Mechanical Engineers [7] has also issued tables, but these are based on an alternative, less accurate formulation, although it is speedier for computation. None of these sources tabulates ice properties. The author has prepared tables of the less common properties for both the solid and the saturated states from  $-20\text{ °C}$  to  $50\text{ °C}$ , an exception being made in that values of the specific enthalpy (included in view of the need for its values in heat transfer) and saturation vapor pressure are also given<sup>1</sup>. These values appear in Table 1. Simple equations fitted to these property values are given in Table 2. By restricting the fittings to modest temperature ranges their accuracy is probably close to that of the tabulated values. For polynomial fittings for the compressed liquid based upon [1], the reader is referred to [12].

## References

- [1] L. Haar, J. S. Gallagher, G. S. Kell, *NBS/NBC Steam Tables* (Hemisphere, Washington, DC, 1984).
- [2] U. Grigull, J. Straub, P. Schiebener, *Steam Tables in SI Units* (Springer-Verlag, Berlin, 1984).
- [3] A. A. Aleksandrov, M. S. Trakhtenherz, *Thermophysical Properties of Water at Atmospheric Pressure* (Standartov, Moscow, 1977).
- [4] W. A. Walker, *Thermodynamic Properties of Liquid Water to One Kilobar* (US Naval Ordnance Laboratory, White Oak, MD, report NOLTR 66-217 ASTTA Doc. 651105, 1967).
- [5] A. H. Harvey, *Thermodynamic Properties of Water: Tabulation from the IAPWS Formulation 1995 for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use* (National Institute of Standards and Technology, Boulder, CO, report NISTIR 5078, 1998).
- [6] W. Wagner, A. Pruss, 'The IAPWS formulation 1995 for the thermodynamic properties of ordinary water substance for general and scientific use', *J Phys Chem Ref Data*, **31**(2) (2002), 387-535.
- [7] W. T. Parry, J. C. Bellows, J. S. Gallagher, A. H. Harvey, *ASME International Steam Tables for Industrial Use* (ASME publ. CRTD, vol. 58, American Society of Mechanical Engineers, New York, 2000).
- [8] *Revised Release on the IAPS Formulation 1985 for the Thermal Conductivity of Ordinary Water Substance* (International Association for the Properties of Water and Steam, London, 1998).
- [9] *Revised Release on the IAPS Formulation 1985 for the Viscosity of Ordinary Water Substance* (International Association for the Properties of Water and Steam, Erlangen, 1997).
- [10] S. Fukusako, 'Thermophysical properties of ice, snow and sea ice', *Int. J. Thermophys.*, **11**(2) (1990), 353-372.
- [11] *IAPWS Release on the Surface Tension of Ordinary Water Substance* (Palo Alto, California, 1994).
- [12] P. E. Liley, 'Accurate simple equations for liquid water properties', *Int. J. Mech. Enging. Educ.*, **27**(4) (1999), 317-323.

<sup>1</sup> A appendix A of [5] reproduces the IAPWS release of September 1996 in which the numbers in [5], which include those for saturation vapor pressure and enthalpy, are based. The release details the formulation which is a fundamental equation for the specific Helmholtz free energy. Relations are given by which individual thermodynamic properties can be derived from the Helmholtz function.