Design and manufacture of screw threads

Nzumbe-Mesape Ntoko
University of Buea, PO Box 63, Buea, Cameroon
E-mail: nmntokosr@yahoo.fr

Abstract It is shown that the profile of a screw thread, a helix, is also the trajectory of a particle of constant axial speed in a rotating coordinate system. Similitude between the motion of the particle and that of the tip of the cutting tool on a cylindrical blank rotated by the chuck of a lathe provides the basis for obtaining cutting speeds from the equation of a helix. The M20 bolt is used as an example.

Keywords helix; geometry of screw threads; cutting speed

Notation

D	diameter (mm)
$\boldsymbol{i, j, k}$	Cartesian unit vectors
N	rotational speed (rpm)
p	pitch of a screw thread (mm)
$\bar{\boldsymbol{r}}$	position vector
$\dot{\boldsymbol{r}}$	velocity vector
$\hat{\boldsymbol{r}}$	radial unit vector
R	radius (mm)
t	time (s)
V_c	cutting speed (m/min)
V_z	axial component of velocity (m/s)
x, y, z	Cartesian coordinate axes
z	axial distance
β	helix angle (rad)
ϕ	polar angle (rad)
$\dot{\phi}$	angular velocity (rad/s)
ω	angular velocity (rad/s)
$\boldsymbol{\varpi}$	angular velocity vector
$\hat{\theta}$	unit tangent vector

Introduction

A helix is the locus of a point which moves with constant speed on a cylindrical surface in a direction that has a constant inclination to the axis. The component of velocity along the axis of the cylinder is constant. The axial distance covered for every revolution is the pitch of the helix.

Helical features characterise all screws and bolts (Fig. 1) as well as some gears. For screws and bolts, the feature is the thread; and for helical gears, the tooth. These features have a helical profile which is defined by a helix angle, the angle made by

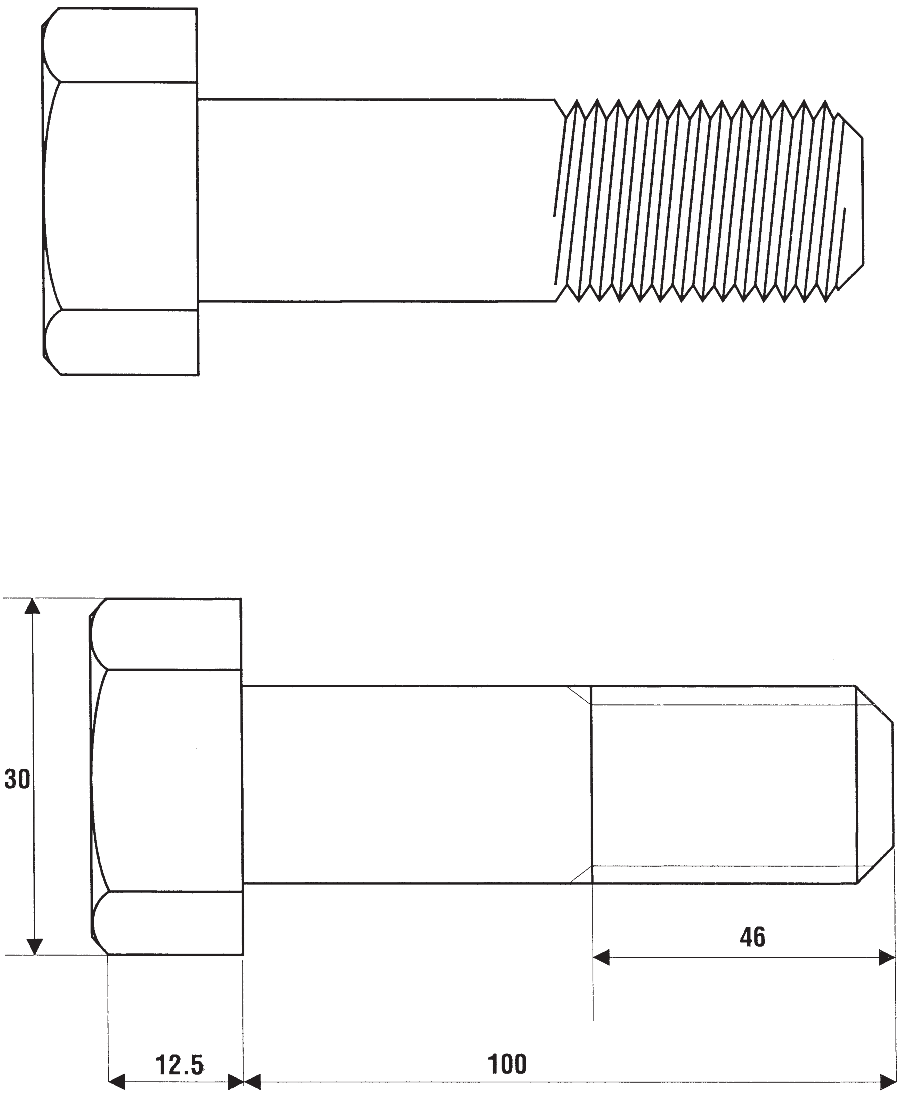

Fig. 1 *An H.M20-100 bolt.*

the tangent to the profile and the axis of the component. Screw threads, depending on the application, may have triangular, trapezoidal or rectangular sections. In all cases the curves defining the screw thread in theory generate ruled surfaces.

While screws and bolts are used to assemble components, gears are used to transmit motion. These components therefore play important roles in mechanical construction and are studied in undergraduate mechanical engineering. Their representations in an assembly would be treated in courses on engineering graphics, their performance in courses on mechanics and machines, and their production, to some extent, in workshop practice.

The present paper introduces the equivalence of helixes and the trajectory of a particle of constant axial speed as seen by an observer in a rotating coordinate system. Hence the feature produced when a blank is rotated by a chuck at constant rotational speed (rotating coordinate system) in contact with the tip of a cutting tool (the particle) moving in the axial direction at constant speed is a helix. It is therefore understandable why the equation of a helix is used to obtain the cutting speed of a screw.

The present contribution may be useful to workshop instructors, who tend to lay more emphasis on practice than on design principles. It is also introductory in nature in the general area of computer-aided geometric design, where the mathematical definition of curves and surfaces has played an important role in industry for over 20 years in design and manufacture.

Equation of a helix

If p is the pitch and ϕ the polar angle (Figs 2 and 3), the position vector, $\bar{\boldsymbol{r}}$, of a variable point on a helix is given by:

$$\bar{\boldsymbol{r}} = R(\cos\phi\boldsymbol{i} + \sin\phi\boldsymbol{j}) + \frac{\phi p}{2\pi}\boldsymbol{k} \qquad (1)$$

where R is the radius of the cylindrical surface, and $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit Cartesian vectors. The unit vector $\boldsymbol{k}$ is in the direction of the axis of the cylinder. Obviously, the polar angle, ϕ, is given by:

$$\phi = \frac{2\pi z}{p} \qquad (2)$$

where z is the axial position of a variable point on the helix.

In a first course on vector algebra, the following relationships between the Cartesian unit vectors ($\boldsymbol{i}, \boldsymbol{j}$) and the cylindrical unit vectors ($\hat{\boldsymbol{r}}$, θ) (Fig. 2) would be seen:

$$\hat{\boldsymbol{r}} = \cos\phi\boldsymbol{i} + \sin\phi\boldsymbol{j}$$
$$\hat{\theta} = -\sin\phi\boldsymbol{i} + \cos\phi\boldsymbol{j} \qquad (3)$$

Hence equation 1 also takes the following form:

$$\bar{\boldsymbol{r}} = R\hat{\boldsymbol{r}} + \frac{\phi p}{2\pi}\boldsymbol{k} \qquad (4)$$

Velocity of a point describing a helix

Differentiating equation 4 with respect to time, one obtains:

$$\dot{\bar{\boldsymbol{r}}} = R\frac{\mathrm{d}\hat{\boldsymbol{r}}}{\mathrm{d}t} + \frac{\dot{\phi}p}{2\pi}\boldsymbol{k} = R\varpi \times \hat{\boldsymbol{r}} + \frac{\dot{\phi}p}{2\pi}\boldsymbol{k} \qquad (5)$$

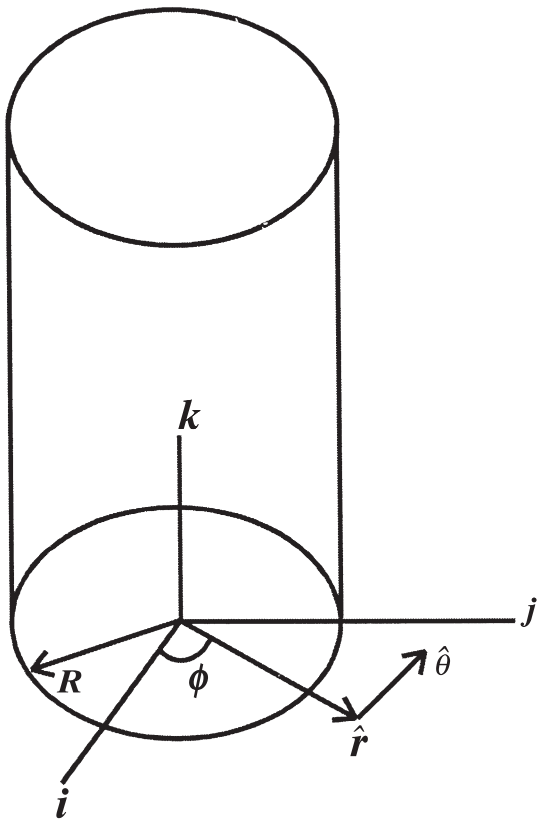

Fig. 2 *Cartesian and polar coordinate systems.*

where the dots denote derivatives with respect to time. The above equation can then simply be written as follows:

$$\dot{\boldsymbol{r}} = \overline{\boldsymbol{V}}_Z + \boldsymbol{\varpi} \times \overline{\boldsymbol{r}} \tag{6}$$

where the velocity vector, $\overline{\boldsymbol{V}}_z$, is given, using equation 2, by:

$$\overline{\boldsymbol{V}}_Z = \frac{\dot{\phi} p \boldsymbol{k}}{2\pi} = \frac{p\boldsymbol{\varpi}}{2\pi} \tag{7}$$

Hence, if the scalars V_z and ω are constant, then the speed of a variable point describing a helix is also constant. Also, equation 6, when written in the form

$$\dot{\boldsymbol{r}} = \omega R \hat{\theta} + V_Z k \tag{8}$$

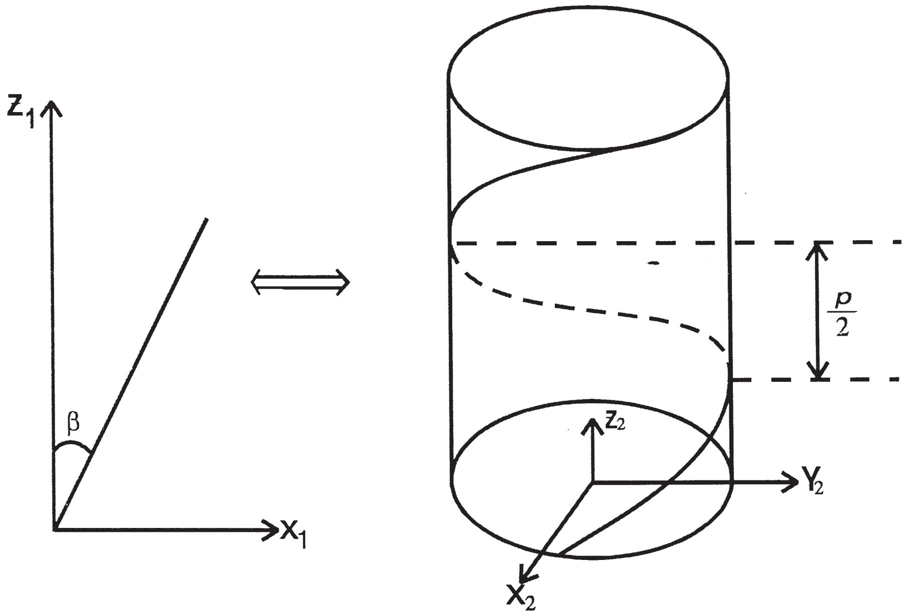

Fig. 3 *A helix and its development.*

yields the following expression for β, the angle of the tangent to the curve with the axis:

$$\cot \beta = \frac{V_Z}{\omega R} = \frac{p}{2\pi R} \tag{9}$$

Equation 9 links the velocity of the cutting tool to the rotational speed of the blank and the pitch of the thread.

Kinematic perspective

With respect to an observer in a rotating coordinate system which rotates about the z-axis with constant angular velocity, ω, equation 6 also defines the velocity of a particle moving with velocity V_z [1]. If that particle represents the tip of a cutting tool on a cylindrical blank rotating with constant angular velocity, $\boldsymbol{\varpi}$, it is evident that the curve traced out by the tip of the tool on the blank is a helix. This is therefore the kinematic basis for producing screws using a lathe. The shape and type of cutter determine the cross-sectional geometry of the thread. Also, equation 8 can be generalised to obtain screws of a conical shape, for example.

The cutting speed

The speed, V_c, of the tip of a cutting tool on the surface of a cylindrical blank is the cutting speed. It is given by equation 8 as follows:

$$V_c = \sqrt{V_z^2 + \omega^2 R^2} \tag{10}$$

Equations 7 and 10 then yield:

$$V_c = \omega\left[\left(\frac{p}{2\pi}\right)^2 + R^2\right]^{\frac{1}{2}} \tag{11}$$

Equation 11 will then be found to yield the following expression for the rotational speed, in revolutions per minute, of a screw blank:

$$N = \frac{60V_C}{\left[p^2 + (2\pi R)^2\right]^{\frac{1}{2}}} \tag{12}$$

As the screw pitch, p, and blank radius, R, are usually given in millimeters and the cutting speed in metres per minute, the above expression becomes the following:

$$N = \frac{1000V_C}{\left[p^2 + (2\pi R)^2\right]^{\frac{1}{2}}} \tag{13}$$

Also, as the pitch, p, is usually much smaller than the radius of the blank, R, the above equation has the following approximate form:

$$N = \frac{1000V_c}{\pi D} \tag{14}$$

where D is the diameter of the blank.

Calculation of rotational speed

For bolts made of mild and carbon steels, a cutting speed in the range 16–20 m/min is recommended [2] when using a high-speed tool. For some alloy steels the optimum cutting speed is in the range 13–16 m/min using such a tool. For carbide-tipped tools the corresponding speeds are about five times those for high-speed tools.

To produce an M20 bolt, for example, in a teaching environment the lower value of the cutting speed for a given material will be chosen for the tool available. That value will then be used to calculate the rotational speed of the blank. Also, as the pitch of the bolt is normally 2.5 mm, equation 13 yields a rotational speed of 254 rpm. Equation 14 produces a result which is accurate to less than 0.1%, which is sufficient.

It is noted that this rotational speed is the optimum for chip removal and cost of production [2]. It is determined by factors such as tool life, cutting forces and surface speed. In a teaching environment, therefore, lower speeds with good chip formation and removal could be used. Lower speeds can prevent accidents such as the tool crashing into the chuck.

Conclusion

It is hoped that the above will show the importance of basic concepts in some undergraduate and professional programmes in design and manufacture which lay more emphasis on practice. Also, because analytical curves serve as an introduction to computer-aided geometric design, the present work can be used as an example on such a course, especially as cutting speed can be linked to the tangent vector of a helix.

References

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[2] R. Dietrich, *et al*., *Précis de Méthodes d'Usinage*, 5th edn (Nathan, Paris, 1981).