An efficient Monte Carlo approach for determining shape factors

Mohammad Biglarbegian
Mechanical and Industrial Engineering Department, University of Toronto, Canada
E-mail: mobiglar@yahoo.com

Abstract In this paper, the Monte Carlo method is used to determine the shape factor among some simple surfaces. The results have been compared with the exact solutions. The results will lead to an efficient method for determining shape factors for some complex surfaces as well.

Keywords radiation; shape factor; Monte Carlo; numerical methods; arbitrary surfaces

Introduction

The shape factor between two surfaces, used for radiation heat transfer, strictly depends on the geometry and orientations of those surfaces to each other. Various methods have been developed and used for determining the shape factors. In some simple cases, exact formulas have been derived.

However, real engineering problems usually involve complicated geometry and require new methods for solution. One of the most efficient commonly used solutions of this kind of problem is the Monte Carlo method. This method, which has been proposed for the solution of radiant exchange in an enclosure, has become by far the most popular, due to its convenient handling of geometrically complex problems. The Monte Carlo method is a numerical solution to a problem that models surfaces interacting with other surfaces or their environment based upon simple surface–surface or surface–environemnt relationships [1]. The use of the Monte Carlo method in radiation heat transfer goes back as far as the paper by Howell and Perlmutter [2]. Many researchers have devised Monte Carlo based schemes. In this paper, the Monte Carlo method is used for calculating the radiation shape factor.

The Monte Carlo method uses random variables. These are used to define ray saved points and ray direction. The number of rays that have been intersected by the second surfaces are counted. Dividing these by the total number of rays emitted through the first surface, the shape factor between the surfaces can be easily found, without complicated algebra. In addition to this, the effect of any obstacle between the surfaces can be easily determined, by counting the number of the rays that have intersected the second surface but excluding those that have intersected the obstacle. Traditional, computational methods would take considerable amount of time. In some cases, it is impossible to use these methods because of the complex orientation of the surfaces to each other.

The present paper reports a computer program that use the Monte Carlo technique for determining shape factors between two parallel plane surfaces, two perpendicular planes, and two parallel disks. The results have been compared with the exact solutions found in a classic heat transfer textbook [3].

The problem

Consider two arbitrary surfaces which have arbitrary orientation relative to each other (Fig. 1). The shape factor which is that fraction of the radiant energy that has been emitted by the first one and received by the second one is given by:

$$F_{ij} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \mathrm{d}A_i \mathrm{d}A_j$$

The notation is explained in Fig. 1. As is clear from this formula, the determination of the double integral will be especially difficult for complicated orientations or surfaces.

Here, it is intended to use Monte Carlo method to determine the shape factor between two parallel planes, two perpendicular planes, and two parallel disks. For these situations, because of the simplicity of the geometry, the above integral for each case has been exactly calculated for comparison with the Monte Carlo results.

Procedure

For a set of random numbers that have been distributed uniformly throughout a surface, different rays in random directions are sent by random selection of angles θ and Φ with respect to the local coordinate. Two random numbers, $0 < R_\theta < 1$ and $0 < R_\Phi < 1$, are selected, and θ and Φ are then calculated using $\theta = \sin^{-1}\sqrt{R_\theta}$ and $\Phi = 2\pi R_\phi$

The number of rays that intersect the second surface are counted. Then, by dividing the number of rays that have intersected the second surface by the total number of rays emitted through the first surface, the shape factor between the surfaces is found, without the need for complicated algebra.

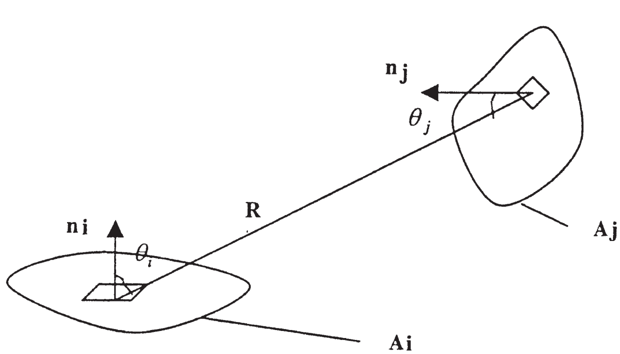

Fig. 1 *Two arbitrary surfaces at an arbitrary orientation relative to each other.*

Two parallel planes

For two parallel planes, as shown in Fig. 2, the exact shape factor can be calculated by:

$$\bar{X} = \frac{X}{L}, \bar{Y} = \frac{Y}{L}$$

$$F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}} \left\{ \mathrm{Ln}\left[\sqrt{\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}} \right] + \bar{X}\sqrt{(1+\bar{Y}^2)}\tan^{-1}\frac{\bar{X}}{\sqrt{(1+\bar{Y}^2)}} \right.$$

$$\left. + \bar{Y}\sqrt{(1+\bar{X}^2)}\tan^{-1}\frac{\bar{Y}}{\sqrt{(1+\bar{X}^2)}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y} \right\}$$

(See Fig. 2 for notation.) For $X = 10$, $Y = 5$, $L = 4$, then $F_{ij} = 0.3559$. As can be seen from Fig. 3, when the number of rays (random numbers) increases, the shape factor obtained from Monte Carlo method converges to the exact solution.

Two perpendicular planes

The situation with the two perpendicular planes is sketched in Fig. 4. The exact solution is:

$$H = \frac{Z}{X}, W = \frac{Y}{X}$$

$$F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - \sqrt{H^2+W^2}\tan^{-1}\frac{1}{\sqrt{H^2+W^2}} \right.$$

$$\left. + \frac{1}{4}\mathrm{Ln}\left\{ \frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(1+H^2)}\right] \times \left[\frac{H^2(1+H^2+W^2)}{(1+W^2)(H^2+W^2)}\right]^{H^2} \right\} \right)$$

For $X = 10$, $Y = 8$, $L = 5$ then $F = 0.1746$.

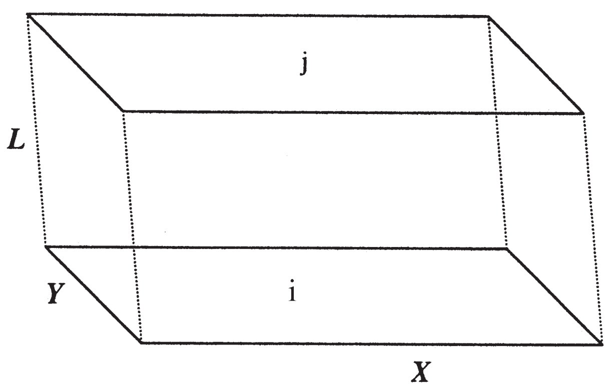

Fig. 2 *The two parallel planes.*

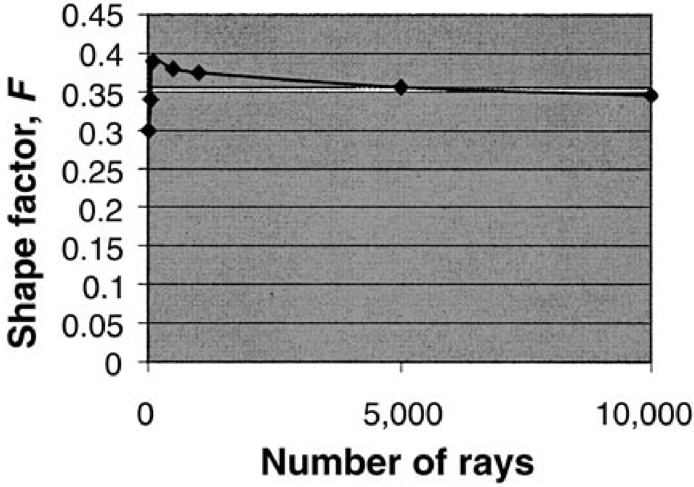

Fig. 3 *Shape factor,* F, *derived for two parallel planes, using the Monte Carlo method. The* x-*axis plots the number of rays used in the calculation.*

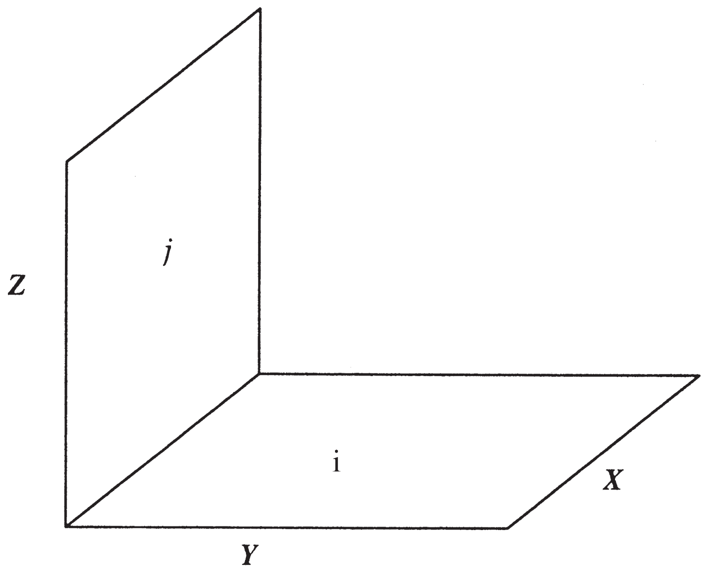

Fig. 4 *The two perpendicular planes.*

Clearly, the Monte Carlo results are satisfactory when the number of rays (random numbers) increases, and for higher number of rays we would have very accurate shape factors. This is shown in Fig. 5.

Two parallel disks

The situation with the two parallel disks is sketched in Fig. 6. The exact solution is:

$$R_i = \frac{r_i}{L}, R_j = \frac{r_j}{L}$$

$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

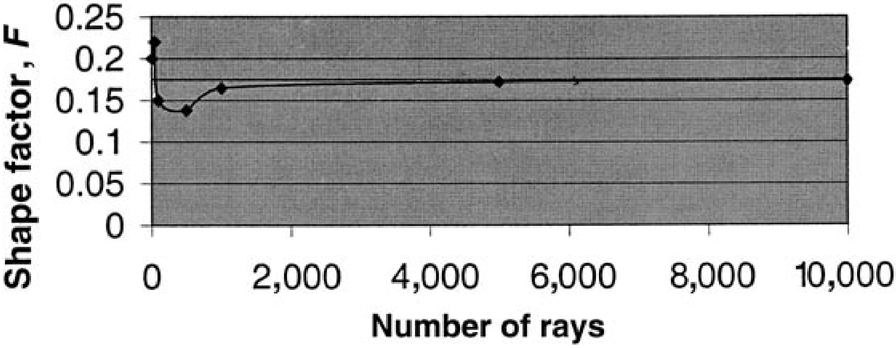

Fig. 5 *Shape factor,* F*, derived for two perpendicular planes, using the Monte Carlo method. The* x*-axis plots the number of rays used in the calculation.*

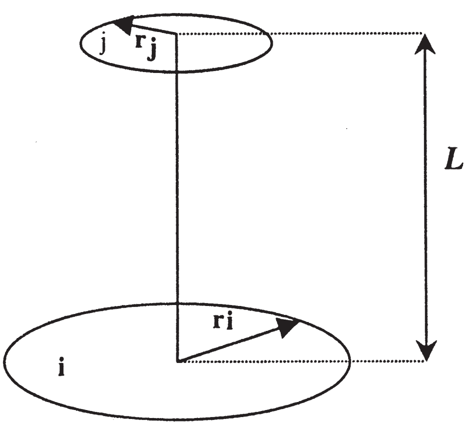

Fig. 6 *The two parallel disks.*

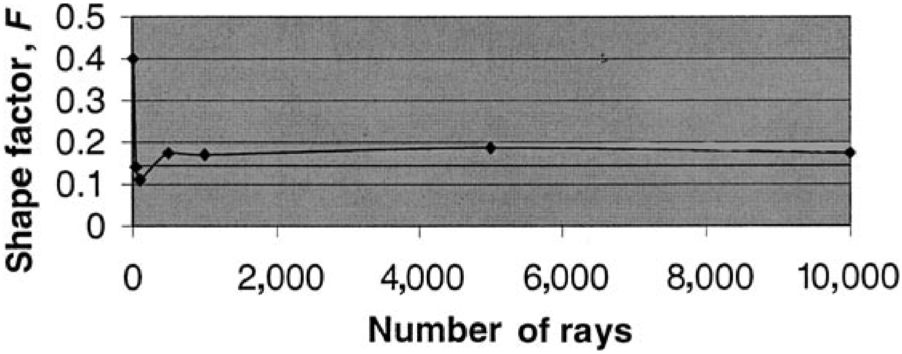

Fig. 7 *Shape factor,* F, *derived for two parallel disks, using the Monte Carlo method. The* x-*axis plots the number of rays used in the calculation.*

$$F_{ij} = \frac{1}{2}\left\{S - \sqrt{s^2 - 4\left(\frac{r_j}{r_i}\right)^2}\right\}$$

For $r_i = 10$, $r_j = 5$, $L = 8$ then $F = 0.1431$.

Again, as the number of rays increase, Monte Carlo results converge to the exact solution (Fig. 7).

Conclusion

The Monte Carlo method was used to determine the shape factor between some simple surfaces. From Figs 3, 5 and 7, it is easily understood that, as the number of rays increases, we would get good results and the difference between the exact solution and this method becomes negligible. Therefore, it seems to be one of the most efficient methods for determining the shape factors between some complex surfaces.

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