Quick-return mechanism design and analysis projects

Ron P. Podhorodeski, Scott B. Nokleby and Jonathan D. Wittchen
Robotics and Mechanisms Laboratory, Department of Mechanical Engineering, University of Victoria, PO Box 3055, Victoria, British Columbia, Canada, V8W 3P6
E-mail: podhoro@me.uvic.ca; snokleby@me.uvic.ca; jwittchen@crsrobotics.com

Abstract Quick-return (QR) mechanisms feature different input durations for their working and return strokes. The time ratio (*TR*) of a QR mechanism is the ratio of the change in input displacement during the working stroke to its change during the return stroke. Several basic types of mechanism have a QR action. These types include slider-crank and four-bar mechanisms. A project on QR mechanism design, within a first course on the theory of mechanisms, has been found to be effective for exposing students to concepts of mechanism design and analysis. This paper reviews basic QR mechanisms, presents a project problem and solution examples, and discusses the value of inclusion of such project problems within theory-of-mechanism courses.

Keywords mechanism projects; design; analysis; synthesis

Introduction

Quick-return mechanisms

Quick-return (QR) mechanisms feature different input durations for their working and return strokes. The time ratio (*TR*) of a QR mechanism is the ratio of the change in input displacement during the working stroke to its change during the return stroke. QR mechanisms are used in shapers, power-driven saws, and many other applications requiring a load-intensive working stroke in comparison to a low-load return stroke [1–3].

Several basic types of mechanism have a QR action. These include slider-crank mechanisms (e.g., see the offset slider-crank mechanism in Fig. 1a and the inverted slider-crank mechanisms, including the crank-shaper mechanism, in Fig. 1b and the Whitworth in Fig. 1c) and four-bar mechanisms (e.g., see the crank-rocker-driven piston in Fig. 2a and the drag-link-driven piston in Fig. 2b).

Mechanism analysis techniques taught in a first course on the theory of mechanisms can be applied to evaluate the performance of QR mechanisms. Design of a mechanism, on the other hand, requires determining a mechanism to perform a desired task. For example, synthesis of a reciprocating QR device requires determination of a mechanism to produce a desired *TR* and a necessary stroke. Note that there is not necessarily a unique mechanism design for a particular task: many mechanism types (e.g., offset slider-crank, Whitworth, drag-link, etc.) may be capable of performing it. Even within one mechanism type, many different link-length combinations (perhaps an infinity of several dimensions [1]) may perform the required task.

Choosing a type of mechanism for a task is called type synthesis. Selecting link lengths for a chosen type is referred to as dimensional synthesis [1–3]. When many

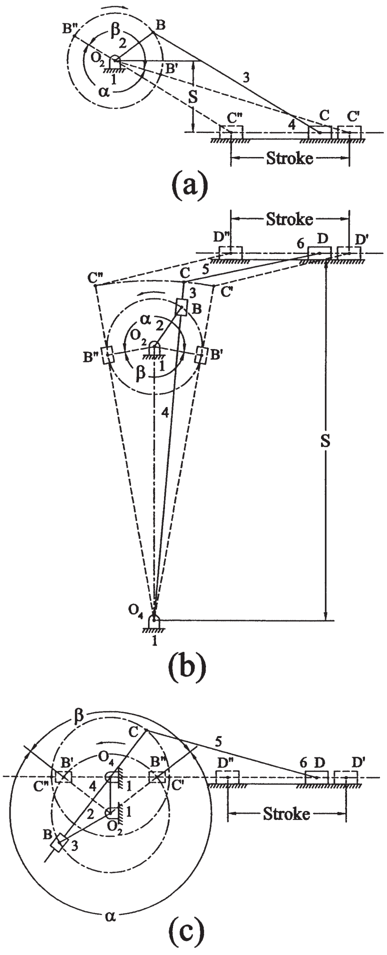

Fig. 1 *Slider-crank QR mechanisms: (a) offset slider-crank, (b) crank-shaper, (c) Whitworth.*

mechanisms of various types and/or dimensions that satisfy the primary task exist, concerns such as mechanism size, minimum transmission angles, maximum accelerations, etc., can be considered to isolate a preferred design.

The task of a QR mechanism is simple to understand. Several concepts of design and analysis can be illustrated by a QR mechanism project. For example, students can be exposed to concepts of kinematic analysis, of minimum transmission angles, of type and dimensional synthesis, and of computer-aided modelling programs.

Several techniques can be considered and developed by students to achieve the required synthesis task; for example, physical modelling, graphical, iterative, and analytical techniques can all be used to synthesize a desired mechanism. Having a laboratory manual that briefly outlines different possible techniques, and leaves the

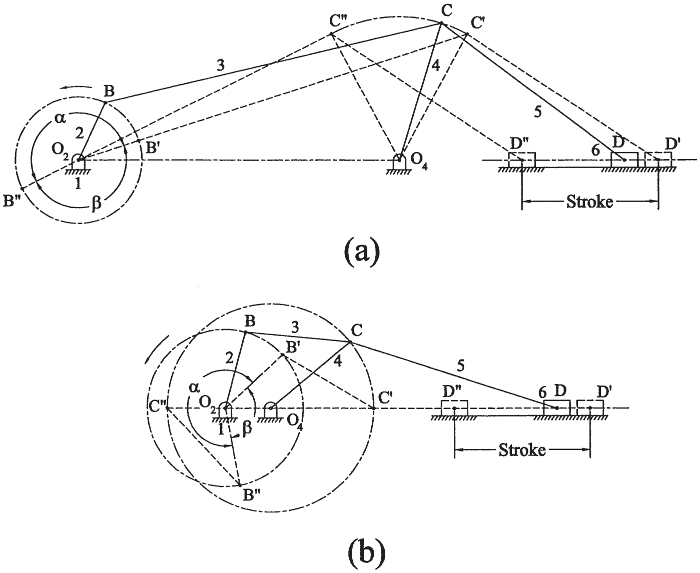

Fig. 2 *Four-bar QR mechanisms: (a) crank-rocker as the driving mechanism, (b) drag-link (crank-crank) as the driving mechanism.*

student-applied technique open, requires a creative algorithm-design process. Over the past 10 years at the Department of Mechanical Engineering, University of Victoria, a variety of projects featuring different mechanism types have been used within a first course on the theory of mechanisms. The QR project, along with similar technique-open projects on inertia modelling and on cam design, has given the students a strong appreciation of mechanism analysis and design issues, and has allowed the assignment to the course of a significant percentage of accreditation units (AUs) for Engineering design [4].[1]

The project described in this work is assigned to and completed by the students within the first four weeks of a first course on mechanism analysis. This course occurs in the first term of third year, of a semestered four-year academic programme that leads to an accredited bachelor of engineering in mechanical engineering degree.

---

[1] The Canadian Engineering Accreditation Board (CEAB) performs accreditation of all undergraduate engineering programmes in Canada. AUs are assigned to the curriculum content of the courses within the program under consideration. Currently AUs are divided between (a) mathematics, (b) basic sciences, (c) engineering sciences, (d) engineering design, and (e) complementary studies. Quoting CEAB Accreditation Criteria and Procedures [4]: ‘Engineering design integrates mathematics, basic sciences, engineering sciences and complementary studies in developing elements, systems and processes to meet specific needs. It is a creative, iterative and often open-ended process subject to constraints. . . .’ While not strong on the complementary aspect, the projects are strong on the creative, iterative, open-ended, and subject-to-constraints aspects.

Outline of the content of the remaining sections

First, types of QR mechanisms and potential techniques for their synthesis are reviewed. The subsequent section presents a typical set of requirements for the QR project. Note that the project requires application of analysis techniques taught very early within a first course on the theory of mechanisms, requires the development of relevant synthesis techniques, and exposes students to the application of computer-based algorithms for the analysis of mechanisms. Examples of solution techniques that have been used to solve portions of the QR project are then presented. The paper closes with further considerations and conclusions.

Quick-return mechanism types and synthesis techniques

Example QR mechanisms

Consider the offset slider-crank illustrated in Fig. 1a. The crank (member 2) is rotating clockwise and rotates a displacement α (B′ to B″) as the piston, C, moves from C′ (top-dead-centre, TDC) to C″ (bottom-dead-centre, BDC). As the piston moves from BDC to TDC the crank rotates a displacement β (B″ to B′). The time ratio (*TR*) is given by:

$$TR = \alpha/\beta \tag{1}$$

A crank-shaper is comprised of a tool driven by an inverted slider-crank. The crank length of a crank-shaper is less than the base length (O_2 to O_4) of the mechanism. Fig. 1b illustrates a typical configuration. Notice that the crank (member 2) is rotating counter-clock wise in this case and that the follower (member 4) of the driving mechanism (the inverted slider-crank) oscillates between two extremes. The crank displacements at these extremes define the values of α and β for the device's *TR*.

A Whitworth mechanism (Fig. 1c) is formed when the crank of the slider-crank inversion is greater than the base distance. Fig. 1c illustrates a Whitworth QR mechanism, where again the crank (member 2) is rotating counter-clockwise. Notice that the follower (member 4) of the Whitworth is dragged through a full rotation during a revolution of the crank. The crank displacements when the follower is parallel to the sliding direction (horizontal in Fig. 1c) define the values of α and β.

Fig. 2a shows a piston being driven by the follower of a crank-rocker four-bar linkage. From the oscillation extremes of the follower, the crank positions B′ to B″ are defined. Fig. 2b depicts a QR mechanism driven by a drag-link (also known as a crank-crank) linkage. The extreme positions of the piston occur when the follower direction is parallel to the sliding direction (horizontal in Fig. 2b).

Design of QR mechanisms

After choosing a mechanism type, appropriate dimensions for the desired task must be selected. Several techniques can be applied. The most basic techniques are physical modelling and graphical. In physical modelling, a scale model (e.g. a 'cardboard and pin' model) is made and the output for a given input is directly measured. The graphical technique involves drawing the mechanism in its various positions.

Physical modelling and graphical solutions are time consuming and can be inaccurate. An alternative is to derive analytical expressions for the mechanism lengths required for a desired *TR*. Note, however, that it is not always possible to derive a closed-form solution for link lengths as a function of a desired *TR*, due to the non-linear from of the *TR* solution. However, if a closed-form solution for the displacements of the driving mechanism can be found, a solution of the *TR* for given link lengths can be found iteratively. Searching over the feasible link lengths allows mechanisms having desired *TR*s to be resolved.

An example quick-return project

The idea of this project is to expose students to concepts of mechanism synthesis and to provide a practical problem where analytical, graphical, and computer-aided analysis techniques can be applied. An example project problem, for designing a drag-link-based QR mechanism, is given below. Available for this project are two mechanism analysis programs: GNLINK [5], a program developed at the University of Manitoba and the University of Toronto, and the commercial program Working Model® [6].

It should be noted that any QR mechanism type can be substituted for the presented drag-link-based one. Substituting different mechanism types allows the teaching objectives of the project to remain the same, but allows for modification of the project from year to year.

Example problem background

An application requires a QR mechanism with *TR* = 1.500 and a stroke of 0.300 m. Currently a drag-link-based QR mechanism exists, as illustrated in Fig. 3. The current lengths of the drag-link mechanism are: distance between fixed centres O_2 and $O_4 = r_1 = 0.1000$ m, length of crank $O_2A = r_2 = 0.2250$ m, length of coupler AB $= r_3 = 0.3000$ m, and length of follower $O_4B = r_4 = 0.2750$ m. The length of the slider's coupler is $CD = r_6 = 0.3000$ m and it is connected a distance $O_4C = r_5 = 0.1000$ m from O_4.

The current drag-link crank and follower are made of cast iron and would be expensive to modify. It is proposed to design a new coupler, AB (length r_3), for the four-bar and to relocate pin C (length r_5) on the follower to create a mechanism capable of performing the task requirements. Furthermore, it is suggested that the coupler should be adjustable in length for future modification of the drag-link-based QR for other *TR*s.

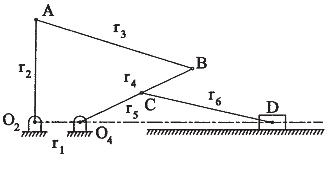

Fig. 3 *Layout of drag-link QR mechanism.*

Example project requirements

Design

(1) Determine a coupler link length (r_3) and C pin location (r_5) satisfying a *TR* = 1.500 and stroke = 0.3000 m while maintaining the other current link lengths.
(2) Determine the range of (r_3) that the adjustable coupler should accommodate to allow the maximum number of drag-link-based *TR*s to be created.

Discussion issues

(1) How many mechanisms providing a specific *TR* are possible if only r_3 is varied?
(2) What is the range of *TR* that would be possible by adjusting the length of the coupler?
(3) How many feasible mechanism solutions would exist for a given *TR* if both the base length, O_2O_4, and the coupler length could be changed?
(4) Discuss the issues you would consider in the isolation of a unique mechanism design.

Analysis

For the mechanism with *TR* = 1.500 and stroke = 0.3000 m:

(1) Evaluate the velocity and acceleration of the slider when $\theta_2 = 60°$, using relative motion analysis (polygons). Use $\omega_2 = 10.0$ rad/s and $\alpha_2 = 0.0$ rad/s^2 for this analysis.
(2) Check this result using the program GNLINK or Working Model®.
(3) Simulate the mechanism for a complete revolution of the input crank.

Examples of quick-return project solutions

Examples of solutions for various portions of the problem set out above are given in this section. The solutions presented are examples of various ways that past students have solved portions of the project.

TR and stroke solutions for known link lengths

The TDC and BDC positions of the piston occur when the follower of the drag-link four-bar mechanism is aligned with the sliding direction. Fig. 4 illustrates these two positions. The following equations can be derived using cosine law:

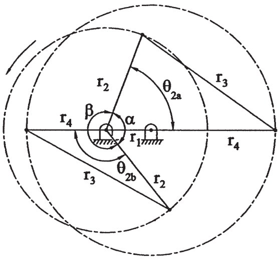

Fig. 4 *Drag-link mechanism at TDC and BDC.*

For TDC: $r_3^2 = r_2^2 + (r_4 + r_1)^2 - 2r_2(r_4 + r_1)\cos(\theta_{2a})$ (2a)

For BDC: $r_3^2 = r_2^2 + (r_4 - r_1)^2 - 2r_2(r_4 - r_1)\cos(\theta_{2b})$ (2b)

Solving for θ_{2a} and θ_{2b} in the expressions for TDC and for BDC yields:

$$\theta_{2a} = \cos^{-1}\left(\left(r_2^2 + (r_4 + r_1)^2 - r_3^2\right)/\left(2r_2(r_4 + r_1)\right)\right) \tag{3a}$$

$$\theta_{2b} = \cos^{-1}\left(\left(r_2^2 + (r_4 - r_1)^2 - r_3^2\right)/\left(2r_2(r_4 - r_1)\right)\right) \tag{3b}$$

In terms of θ_{2a} and θ_{2b}, the duration of the working stroke is:

$$\alpha = \pi - \theta_{2a} + \theta_{2b} \tag{4}$$

Since $\beta = 2\pi - \alpha$, the *TR* can be found as:

$$TR = \frac{\alpha}{\beta} = \frac{\pi - \theta_{2a} + \theta_{2b}}{\pi + \theta_{2a} - \theta_{2b}} \tag{5}$$

The stroke of the drag-link-driven QR mechanism is double the length O_4C, i.e., *stroke* = 2 ∗ O_4C. For the desired stroke of 0.3000 m, O_4C = 0.1500 m.

Iterative solution for the value of r_3

Equation (5), combined with equations (3a) and (3b), is a solution for the *TR* of the device for known link lengths. For the project problem, the feasible range of r_3 can be found considering Grashof's criteria [7] for a drag-link four-bar, i.e.:

$$r_{short} + r_{long} \le r_a + r_b \text{ and } r_{short} = r_1 \tag{6}$$

where r_{short} and r_{long} are the lengths of the shortest and the longest links, r_a and r_b are the lengths of the other two links, and r_1 is the length between the base pins [1–3].

For the given length values, r_{long} will be equal to either r_3 or r_4. With $r_3 = r_{long}$, substituting the known length values into equation (6) yields $0.1 + r_3 \leqslant 0.2250 + 0.2750$ and therefore $r_3 \leqslant 0.40$. Similarly, $r_4 = r_{long}$ yields $0.1 + 0.2750 \leqslant 0.2250 + r_3$ and therefore $0.15 \le r_3$. In summary, for the given values of r_1, r_2, and r_4, values of r_3 in the range 0.1500 m ≤ r_3 ≤ 0.4000 m yield drag-link mechanisms.

Analysis of the values of the *TR* over the feasible r_3 range using the given values of r_1 = 0.1000 m, r_2 = 0.2250 m, r_4 = 0.2750 m yields the *TR* values illustrated in Fig. 5. Searching the *TR* data used to create Fig. 5, two values, r_3 = 0.154 m and r_3 = 0.250 m, are found to yield the desired *TR* = 1.500. Again, examination of the data indicates that *TR* values ranging $1.430 \leqslant TR \le 5.538$ can be achieved, depending on the value of r_3. For a desired *TR*, there are either zero, one, or two feasible solutions for r_3.

Analytical solution for the value of r_3

Solving for θ_{2b} in terms of θ_{2a} and *TR* from equation (5) gives:

$$\theta_{2b} = \theta_{2a} + \frac{(TR - 1)\pi}{TR + 1} = \theta_{2a} + \phi \tag{7}$$

with ϕ being equal to $(TR - 1)\pi/(TR + 1)$.

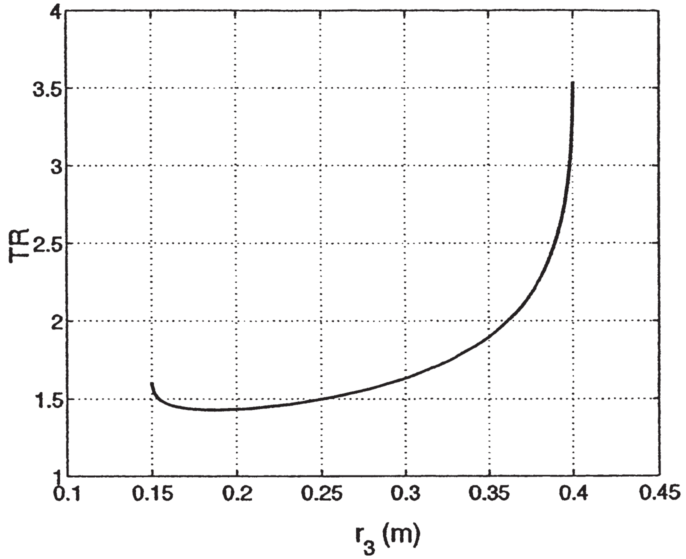

Fig. 5 TR *values for feasible range of* r_3.

Equating the right-hand sides of equations (2a) and (2b) eliminates r_3 and yields:

$$r_2^2 + (r_4 + r_1)^2 - 2r_2(r_4 + r_1)\cos(\theta_{2a}) = r_2^2 + (r_4 - r_1)^2 - 2r_2(r_4 - r_1)\cos(\theta_{2b}) \qquad (8)$$

Cancelling the common r_2^2 term, substituting for θ_{2b} from equation (7), and grouping the cosine terms on the left-hand side gives:

$$2r_2(r_4 + r_1)\cos\theta_{2a} - 2r_2(r_4 - r_1)\cos(\theta_{2a} + \phi) = (r_4 + r_1)^2 - (r_4 - r_1)^2 \qquad (9)$$

Simplifying and using the angle sum relationship for cosine [8], equation (9) becomes:

$$(r_4 + r_1)\cos\theta_{2a} - (r_4 - r_1)[\cos\phi\cos\theta_{2a} - \sin\phi\sin\theta_{2a}] = \frac{2r_4r_1}{r_2} \qquad (10)$$

Letting $A = (r_4 + r_1) - (r_4 - r_1)\cos\phi$, $B = (r_4 - r_1)\sin\phi$, and $C = 2r_4r_1/r_2$ allows equation (10) to be expressed as:

$$A\cos\theta_{2a} + B\sin\theta_{2a} = C \qquad (11)$$

which has the following θ_{2a} solutions [9]:

$$\theta_{2a} = \text{atan2}(B, A) \pm \text{atan2}\left(\sqrt{A^2 + B^2 - C^2}, C\right) \qquad (12)$$

where atan2 (numerator, denominator) denotes a quadrant corrected arctangent function.

From equation (11), if $A^2 + B^2 > C^2$, two solutions for θ_{2a} can be resolved. With θ_{2a} known, equation (2b) can be solved for r_3, i.e.:

$$r_3 = \pm\sqrt{r_2^2 + (r_4 + r_1)^2 - 2r_2(r_4 + r_1)\cos(\theta_{2a})} \tag{13}$$

The negative solutions for r_3 can be neglected since it is physically impossible to have a negative link length. Therefore, if $A^2 + B^2 > C^2$, two feasible solutions for r_3 can exist. The solutions for r_3, however, must be tested to ensure that they satisfy the Grashof criteria for a drag-link four-bar mechanism (Equation (6)). When $A^2 + B^2 - C^2 = 0$, there is only one solution for θ_{2a} and therefore only one potential solution for r_3. When $C^2 > A^2 + B^2$ there is no real solution for θ_{2a} and therefore no solution for r_3. Therefore, there may be zero, one, or two solutions for r_3, depending on the desired *TR* value.

Substituting the specified $TR = 1.500$ and the given length values, we find $\theta_{2a} = 6.18°$ or $39.38°$ and $r_3 = 0.1544\,\text{m}$ or $0.2504\,\text{m}$, respectively. These results confirm the r_3 results found iteratively above.

Selecting the preferred value for r_3

The transmission angle of a mechanism determines the effectiveness it will have in driving its payload. An ideal transmission angle is 90°. A minimum transmission angle of 30° has been suggested [1], as has 45° [2, 3]. In any case, higher transmission angles are preferred, to prevent binding of the links.

The minimum and maximum transmission angles for a drag-link mechanism occur when the follower is aligned with the base link, i.e., for the illustrated drag-link-based QR mechanism the minimum/maximum transmission angles occur at TDC and BDC. Referring to Fig. 6a, the transmission angle at TDC, γ_{TDC}, can be resolved through cosine law, i.e.:

$$r_2^2 = r_3^2 + (r_4 + r_1)^2 - 2r_3(r_4 + r_1)\cos\gamma_{TDC} \tag{14}$$

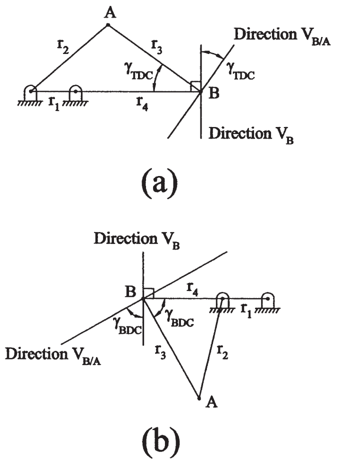

Fig. 6 *Force transmission angles at: (a) TDC and (b) BDC.*

and therefore:

$$\gamma_{\mathrm{TDC}} = \cos^{-1}\left(\frac{r_3^2 + (r_4 + r_1)^2 - r_2^2}{2r_3(r_4 + r_1)}\right) \tag{15}$$

Similarly, with reference to Fig. 6b, BDC yields:

$$r_2^2 = r_3^2 + (r_4 - r_1)^2 - 2r_3(r_4 - r_1)\cos\gamma_{\mathrm{BDC}} \tag{16}$$

giving:

$$\gamma_{\mathrm{BDC}} = \cos^{-1}\left(\frac{r_3^2 + (r_4 - r_1)^2 - r_2^2}{2r_3(r_4 - r_1)}\right) \tag{17}$$

The minimum transmission angle is $\gamma_{\min} = \min(\gamma_{\mathrm{TDC}}, \gamma_{\mathrm{BDC}})$. The known link lengths and the found values, $r_3 = 0.1544$ m and 0.2504 m, yield $\gamma_{\min_{r_3=0.1544}} = \min(10.49°, 85.94°) = 10.47°$ and $\gamma_{\min_{r_3=0.2504}} = \min(35.60°, 60.83°) = 35.60°$. Therefore, $r_3 = 0.2504$ m is the preferred solution.

Discussion issues

(1) As seen by the *TR* calculations for potential r_3 values in Fig. 5, either zero, one, or two r_3 values may exist, depending on the desired *TR* value.

(2) A range of *TR* values, $1.430 \leqslant TR \leq 5.538$, exists for the range of feasible r_3 lengths in drag-link mechanisms.

(3) If both the base length O_2–O_4 and the coupler length, r_3, were adjustable, a single order of infinity of solutions would exist for a given *TR*.

(4) As seen in solving for the preferred value of r_3, the minimum transmission angle can be critical in the isolation of a unique mechanism design.

Analysis of the kinematics

Solving for the slider velocity

Known: $\theta_2 = 60°$, $\omega_2 = 10$ rad/s, $\alpha_2 = 0$ rad/s^2, $O_2A = r_2 = 0.2250$ m, $AB = r_3 = 0.2504$ m, $O_4B = r_4 = 0.2750$ m, $O_4C = r_5 = 0.1500$ m, and $CD = r_6 = 0.3000$ m. Fig. 7a depicts the mechanism with the link lengths required to achieve a *TR* = 1.500.

Using relative velocity analysis:

$$\vec{\mathbf{V}}_{\mathrm{B}} = \vec{\mathbf{V}}_{\mathrm{A}} + \vec{\mathbf{V}}_{\mathrm{B/A}} \tag{18}$$

$$\vec{\mathbf{V}}_{\mathrm{D}} = \vec{\mathbf{V}}_{\mathrm{C}} + \vec{\mathbf{V}}_{\mathrm{D/C}} \tag{19}$$

The relative velocity equations above are solved in sequence and each has two unknown quantities. Table 1 describes the known and unknown vector components for the equations. In Table 1 the symbol ⊥ is used to denote perpendicular and the symbol ? is used to denote an unknown quantity.

The unknowns may be resolved using the graphical method shown in Fig. 7b. Table 2 summarizes the magnitudes found from Fig. 7b.

Solving for the slider acceleration

Using relative acceleration analysis:

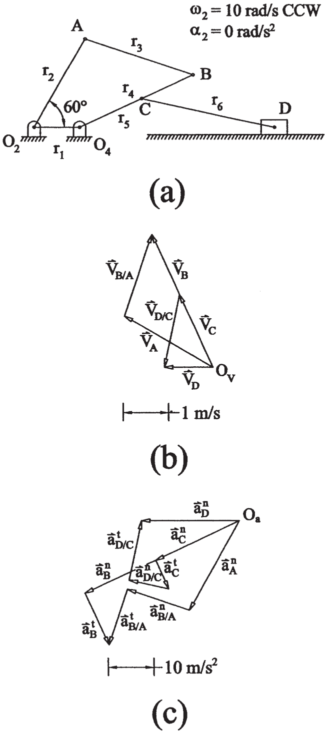

Fig. 7 *Analysis of kinematics at $\theta_2 = 60°$: (a) required drag-link QR mechanism, (b) velocity polygon and (c) acceleration polygon.*

TABLE 1 *Known and unknown velocity components*

Velocity	Magnitude (m/s)	Direction
$\vec{\mathbf{V}}_{A}$	$(\lvert\omega_2\rvert)O_2A = 2.250$	150° (i.e., ⊥ to $\overline{O_2A}$)
$\vec{\mathbf{V}}_{B/A}$	?	⊥ to $\overline{AB}$
$\vec{\mathbf{V}}_{B}$	?	⊥ to $\overline{O_4B}$
$\vec{\mathbf{V}}_{C}$	Found by velocity image	Same direction as $\vec{\mathbf{V}}_{B}$
$\vec{\mathbf{V}}_{D/C}$	?	⊥ to $\overline{DC}$
$\vec{\mathbf{V}}_{D}$	?	Horizontal (sliding direction)

$$\vec{\mathbf{a}}_{B} = \vec{\mathbf{a}}_{A} + \vec{\mathbf{a}}_{B/A}$$

$$\vec{\mathbf{a}}^{n}_{B} + \vec{\mathbf{a}}^{t}_{B} = \vec{\mathbf{a}}^{n}_{A} + \vec{\mathbf{a}}^{t}_{A} + \vec{\mathbf{a}}^{n}_{B/A} + \vec{\mathbf{a}}^{t}_{B/A} \tag{20}$$

$$\vec{\mathbf{a}}_{D} = \vec{\mathbf{a}}_{C} + \vec{\mathbf{a}}_{D/C}$$

$$\vec{\mathbf{a}}^{n}_{D} + \vec{\mathbf{a}}^{t}_{D} = \vec{\mathbf{a}}^{n}_{C} + \vec{\mathbf{a}}^{t}_{C} + \vec{\mathbf{a}}^{n}_{D/C} + \vec{\mathbf{a}}^{t}_{D/C} \tag{21}$$

The relative acceleration equations above are solved in sequence and each has two unknown quantities. Table 3 describes the known and unknown vector com-

TABLE 2 *Velocity magnitudes*

Velocity	Magnitude (m/s)
$\vec{\mathbf{V}}_{B/A}$	1.898
$\vec{\mathbf{V}}_{B}$	3.219
$\vec{\mathbf{V}}_{C}$	1.756
$\vec{\mathbf{V}}_{D/C}$	1.631
$\vec{\mathbf{V}}_{D}$	1.075

TABLE 3 *Known and unknown acceleration components*

Acceleration	Magnitude (m/s^2)	Direction
$\vec{\mathbf{a}}_A^n$	$\omega_2^2 * O_2A = 22.50$	// to O_2A (directed towards O_2)
$\vec{\mathbf{a}}_{B/A}^n$	$\lVert \vec{\mathbf{V}}_{B/A} \rVert^2/AB = 14.39$	// to $\overline{AB}$ (directed towards A)
$\vec{\mathbf{a}}_{B/A}^t$	?	$\perp$ to $\overline{AB}$
$\vec{\mathbf{a}}_B^n$	$\lVert \vec{\mathbf{V}}_B \rVert^2/O_4B = 37.68$	// to $\overline{O_4B}$ (directed towards O_4)
$\vec{\mathbf{a}}_B^t$	?	$\perp$ to $\overline{O_4B}$
$\vec{\mathbf{a}}_C$	Found by acceleration image	Same direction as $\vec{\mathbf{a}}_B$
$\vec{\mathbf{a}}_{D/C}^n$	$\lVert \vec{\mathbf{V}}_{D/C} \rVert^2/CD = 8.87$	// to $\overline{CD}$ (directed towards C)
$\vec{\mathbf{a}}_{D/C}^t$	?	$\perp$ to $\overline{CD}$
$\vec{\mathbf{a}}_D^t$	?	Horizontal (sliding direction)

ponents for the equations. For Table 3 it has been noted that the magnitude of $\vec{\mathbf{a}}_A^t$ is zero since $\alpha_2 = 0\,\text{rad/s}^2$ and that the magnitude of $\vec{\mathbf{a}}_D^n$ is also zero since the piston *D* slides on a straight surface.

The unknowns may be solved for using the graphical construction shown in Fig. 7c. Measuring from Fig. 7c, the magnitude of $\vec{\mathbf{a}}_D^n$ is 21.7 m/s^2, with its direction being to the left.

Computer-aided analysis

The drag-link-driven QR mechanism is a two-loop, single-input mechanism. Vectors can be used to represent the mechanism's links, as illustrated in Figure 8a. With this model, the angular displacement, velocity, and acceleration of vector 2 are the mechanism's inputs.

The mechanism has been simulated on GNLINK and Working Model®. Results from GNLINK are presented. Figure 8b shows a simulation of the mechanism through a full rotation. Table 4 summarizes the vector model used to model the mechanism. Table 5 presents kinematic results output for $\theta_2 = 60°$. The results are identical, to three significant figures, to those found graphically using AutoCAD® above (Fig. 7).

TABLE 4 *GNLINK model of the drag-link QR mechanism*

Initial Vector Information

Vector No.	Length	Angle
1	.1000E+00	.0000E+00
2	.2250E+00	.6000E+02
3	.2504E+00	−.3000E+02
4	.2750E+00	.3000E+02
5	.1500E+00	.3000E+02
6	.3000E+00	.1600E+03
7	.3000E+00	.0000E+00

Dependent Variables

Dependent Variable No.	Vector No.	A or L
1	3	A
2	4	A
3	6	A
4	7	L

Common Variable Pairs

Pair No.	Primary	Secondary	A or L	Difference
1	4	5	A	.0000E+00

Loop Sequences

Loop No.	Sequence			
1	2	3	–4	–1
2	5	–6	–7	

TABLE 5 *GNLINK kinematic results for $\theta_2 = 60°$*

```
Results are tabulated using the following format. Print out may be paused by pressing <Ctrl S>. To
start print out press <CR>.

TIME
   In1              In1D          In1DD          Dep 1          Dep 1D         Dep 1DD
   Dep 2            Dep 2D        Dep 2DD        Dep 3          Dep 3D         Dep 3DD
   Dep 4            Dep 4D        Dep 4DD

T = .0000E+00
   .1047E+01        .1000E+02     .0000E+00      −.3247E+00     .7566E+01      −.5118E+02
.4313E+00           .1169E+02     −.4541E+02     .2931E+01      −.5430E+01     .4400E+02
.4296E+00           −.1074E+01    −.2167E+02
```

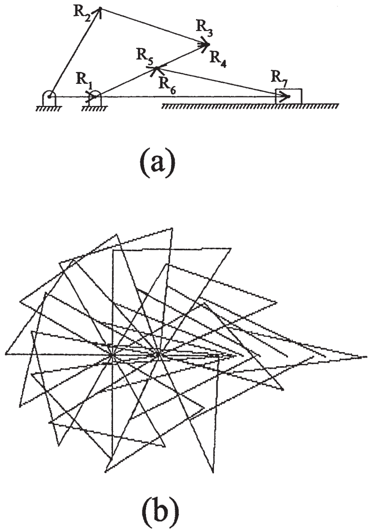

Fig. 8 *GNLINK results: (a) vector model of the drag-link QR mechanism, (b) motion simulation of the mechanism.*

Further considerations

On the accuracy of the graphical velocity and acceleration solutions

The high accuracy achieved in the graphical analysis of velocity and acceleration values is due to the use of a computer-aided drawing package. While not mandatory, approximately 60% of the students utilize such packages and achieve similar accuracy.

On theory of mechanisms laboratory and the format and value of the labs

The laboratory used for the theory of mechanisms class has a reconfigurable mechanism testbed, which allows the construction and running of different mechanism types, including the drag-link-based QR. In addition, the laboratory has several PC-based computers running mechanism simulation software, including GNLINK, the program used in this work, and CAMPRF [10] a cam-profile-design program. Also found in the laboratory are a cut-away five-speed manual transmission and various scales and knife edges to allow the determination of the inertia properties of links.

Students are divided into groups of three for the laboratories. Each group has access to the laboratory for approximately one hour per week. Over the 13 weeks of the term, students are currently scheduled to complete the following four lab projects:

(1) Design and analysis of a QR mechanism.
(2) Approximate modelling and physical determination of inertia properties.

(3) Design and analysis of cam and follower systems.
(4) Observation and calculation of gear reduction ratios.

The timing of the specific projects coincides with the material coverage in the course.

On the manual for the QR project

The laboratory manual for the QR project basically consists of the information found within the introductory sections to this paper. While the background remains the same, the type of mechanism featured in the project varies from year to year.

Conclusions

Having a project related to QR mechanism design and analysis is very beneficial to the students. The experience familiarizes the students with the terminology of mechanisms, with concepts related to mechanism synthesis, with relative motion analysis, and with techniques and computer programs for the design and analysis of mechanisms. Since the task of a QR mechanism and the kinematic analysis involved in the project are basic, the project functions very well within the first four weeks of a first course on the theory of mechanisms to strengthen student understanding of the taught material.

Acknowledgement

The undergraduate students of the Department of Mechanical Engineering, University of Victoria, are thanked for providing effective feedback on the QR mechanism project.

References

[1] A. G. Erdman and G. S. Sandor, *Mechanism Design – Analysis and Synthesis*, 3rd edn (Prentice Hall, Upper Saddle River, NJ, 1997).
[2] J. E. Shigley and J. J. Uicker, Jr, *Theory of Machines and Mechanisms*, 2nd edn (McGraw-Hill, Toronto, 1995).
[3] H. H. Mabie and C. F. Reinholtz, *Mechanics and Dynamics of Machinery*, 4th edn (John Wiley & Sons, Toronto, 1987).
[4] Canadian Engineering Accreditation Board (CEAB), *Accreditation Criteria and Procedures* (Canadian Council of Professional Engineers, 2000).
[5] R. P. Podhorodeski and W. L. Cleghorn, 'Multiple-loop mechanism analysis using a microcomputer', *Proc. of the 9th Applied Mechanism Conference* (Kansas City, MO, October 1985), pp. V1–V7.
[6] Knowledge Revolution, *Working Model 2D – User's Manual* (1996).
[7] E. A. Dijksman, *Motion Geometry of Mechanisms* (Cambridge University Press, New York, 1976).
[8] D. Zwillinger, ed., *CRC Standard Mathematical Tables and Formulae*, 30th edn (CRC Press, New York, 1996).
[9] J. J. Craig, *Introduction to Robotics – Mechanics and Control*, 2nd edn (Addison-Wesley, Don Mills, ON, 1986).
[10] W. L. Cleghorn and R. P. Podhorodeski, 'Disc cam design using a microcomputer', *Int. J. Mech. Enging. Educ.*, **16**(4) (1988), 235–250.