Letter to the Editor

D. G. Clayton

The angle unit

The need for units (e.g. kg, m, s, degree) arises because numerical values alone are insufficient as a means of description. Units are said to have 'dimensions' or are 'dimensional', as distinct from numbers, which are 'dimensionless' or 'non-dimensional'.

In the writer's opinion the angle unit can be classed only as 'dimensional' and yet, in practice, an angle 'measured in radians' is ambiguously taken as an angle unit, e.g. $\sin(\pi/40\,\text{rad})$, or a number, e.g. $(\pi/40)^3$. This practice hides a multitude of sins, such as the removal of the radian unit without logic – and the unreal description 'measured' in radians, when it is widely known that angles are never measured in radians – indeed, such a system would give insoluble practical difficulties.

The official view of the nature of the angle unit is even more extreme. The 20th Conférence Générale des Poids et Mesures (CGPM) in 1995, as Resolution 8, decided:

> to interpret the supplementary units in the SI, namely the radian and the streradian, as dimensionless derived units, the names and symbols of which may, but need not, be used in expressions for other SI derived units, as is convenient. . . .

Putting it rather unkindly, the oxymoron of a 'dimensionless unit' has been imposed, and the names radian and steradian, and symbols rad and sr, are now 'will o'the wisp' items which may vanish and reappear at will.

Mathematically, the former dimensional units, rad and sr, have been changed into the non-dimensional constant 1.0. The previous units of degree and revolution, for instance, have been changed to non-dimensional constants 0.017453292 and 6.283185307 or thereabouts. In the case of the included angle at the corner of a cube (i.e. a trirectangular trihedron or trirectangle) this solid angle is now a non-dimensional constant 1.570796327 or thereabouts. An angle of '1 degree' is now to be regarded as representing the number 1×0.017453292.

The reason for this rather uncomplimentary letter is to point out that the novel use of an abbreviation for radian measure and an abbreviation for steradian measure would avoid the need for any ambiguity or oxymoron. For example: the radian measure of an angle of 0.6 radians is 0.6, and as radian measure = circular measure = arc length, s,/radius, r, then using the abbreviation $\text{circ}\,\theta$ for the circular measure of angle θ avoids the need to turn a radian unit into the non-dimensional constant 1.0, because the required non-dimensional constant 1.0 is now given by the verifiable relation circ 1 radian = 1.0.

Taking a textbook example, if θ is small and measured in radians, $\sin\theta \approx \theta$. If angle θ is small, this becomes $\sin\theta \approx \text{circ}\,\theta$.

This use of circ θ as an abbreviation for the dimensionless circular measure value of angle θ matches the familiar use of sin θ as the abbreviation for one of several dimensionless triangular measures of angle θ. By this means, dimensionless values are produced without offending academic sensibilities.

A controversial expansion of this treatment by the writer, of apparently little interest, claims to show that the radian and the steradian are unnecessary angle units.