A basis for teaching centrifugal pump characteristics in an undergraduate course in turbomachinery

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Abstract The paper presents a basis for teaching centrifugal pump characteristics in an undergraduate course in turbomachinery. It is shown how the shock and frictional loss coefficients, key factors in the determination of pump characteristics, can be appropriately obtained. The first coefficient is obtained from the difference between the drag at a reference incidence and that at zero incidence, while the second is obtained from correlations for frictional loss in a spiral. The coefficients so obtained were used to obtain the real characteristics of a centrifugal pump impeller.

Keywords centrifugal pump; shock and frictional loss coefficients; real characteristics

Notation

A	constant (m)
A_1	constant depending on the angle subtended by the blade
B	constant (sm^{-2})
B_1	constant depending on the ratio r_{av}/D_h
c	chord length of an aerofoil (m)
C	constant (s^2m^{-5})
C_1	constant depending on the ratio t/D_h
C_D	drag coefficient
D	diameter (m)
g	acceleration due to gravity (ms^{-2})
h	component head loss (m)
H	head loss (m)
i	angle of incidence (degrees)
k	curvature (m^{-1})
K	loss coefficient
L	length of the spine of an impeller passage (m)
N	rotational speed (rpm)
P	pressure (Pa)
ΔP_0	stagnation pressure (Pa)
Q	flow rate (m^3/s)
r	radius of a point on an impeller (m)
Re	Reynolds number
s	pitch of a cascade (distance between 2 successive blade elements) (m)
t	axial width of an impeller passage (m)
U	tangential velocity of a point on a pump impeller (m/s)
V	absolute flow velocity at a point in an impeller (m/s)

W	relative velocity of flow (m/s)
Z	number of vanes in the impeller passage
α	expansion angle in diffuser (degrees)
β	flow angle measured relative to the radial direction (degrees)
δ	angle of deviation (degrees)
θ	angle of bend (degrees)
η	hydraulic efficiency
λ	friction factor
ω	blade angle subtended at the centre of a rotor (degrees)
ρ	density of fluid (kg/m^3)
ϕ	angle ($0 \le \phi \le \pi$) in radians

Subscripts

0	nominal conditions
1	inlet to an impeller or other passage
2	outlet from an impeller or other passage
av	average
cv	closed valve
d	diffuser
f	friction
h	hydraulic
m	mean value
max	maximum value
min	minimun value
r	radial component
R	real value
s	value due to shock
sp	spiral
t	tangential component
th	theoretical value

Superscripts

$'$ value for the blade or vane

Introduction

Shock and the frictional loss coefficients are parameters used to determine the characteristics of a centrifugal pump. However, while correlations abound for determining the frictional loss coefficient [1], the same is not true for the shock loss coefficient. In an undergraduate course in turbomachinery, the nature and order of magnitude of the shock loss coefficient, especially, is often not apparent to students. It is also not apparent to students how these coefficients can be obtained. It is therefore the purpose of this paper to present procedures which are appropriate to the level considered, not only for explaining the nature of the shock loss coefficient but also for calculating the values of the two loss coefficients in order to obtain the real characteristics of a centrifugal pump impeller.

The method for determining the shock loss coefficient is based on the application of the principles of superposition to aerofoil drag. In the present application of the principle, the drag coefficient of a turbomachine blade at any incidence is considered to be the sum of the drag at the zero incidence which is responsible for the profile loss [2] and the contribution due to the shock loss. Exploiting this idea and the relationship between the drag coefficient and stagnation pressure loss [2], an expression is obtained for calculating the shock loss coefficient.

Also, to calculate the frictional loss coefficient, the impeller passage was modelled using equivalent simple geometries. The model therefore considered an impeller passage to be geometrically and kinematically equivalent to a rough diffuser and a smooth spiral. The rough diffuser takes care of the frictional and diffusion losses in the impeller, while the spiral takes care of the losses due to change of direction of the flow. The head loss coefficient is then given by the sum of the coefficients of equivalent simple geometries of the model. Students should not have any difficulty accepting the model, as shock and frictional losses would already have been seen in an earlier course in fluid mechanics (incompressible flow).

Expression for the characteristic of a centrifugal pump

The real characteristic of a pump is obtained by subtracting shock and frictional losses from the Euler head of a pump. It is then given in the form:

$$H_{\mathrm{R}} = \mathrm{A} + \mathrm{B}Q + \mathrm{C}Q^2 \tag{1}$$

In the above, A, B and C are constants; H_{R} is the real head and Q the flow rate. The constants are defined by:

$$A = \frac{(\pi N D_2)^2}{3600\ g} - K_s Q_0^2$$

$$B = -\frac{N \tan \beta_2}{60\ gt} + 2K_s Q_0$$

$$C = -K_s - K_f \tag{2}$$

where N is the impeller rotational speed (rpm), D_2 the impeller diameter, β_2 the outlet angle, t the axial width of the impeller at the periphery and Q_0 the nominal flow rate.

However, in the zone where the flow rate is lower than the nominal value, we propose to take into account the losses due to recirculation. The following expression for the head loss due to recirculation is proposed:

$$h_r = 2K_f (Q - Q_0)^2 \tag{3}$$

The basis of the expression is that recirculation results in flow going through the impeller twice. The new expressions of the coefficients A, B and C in equation 1 for that zone are then the following:

$$A = \frac{(\pi N D_2)^2}{3600\ g} - K_s Q_0{}^2 - 2K_f Q_0^2$$

$$B = -\frac{N \tan \beta_2}{60\, gt} + 2K_s Q_0 + 4K_f Q_0$$

$$C = -K_s - 3K_f \tag{4}$$

Equations 4 are new and are used only in the zone where the flow rate is lower than the nominal value to account for losses due to recirculation.

The shock loss coefficient

Theoretical basis

The nominal flow rate, Q_0, for the shock-free flow corresponds to the pump flow rate at zero incidence (when the fluid relative flow angle, β_1, is equal to the blade angle, β_1'). Hence when the flow rate changes about that value and the incidence changes, there is loss of energy due to the misalignment of the flow direction and the camber line of the impeller vanes. The difference between the drag coefficient at any other incidence and that at zero incidence is attributed to shock loss. This means that the drag coefficient at any incidence has both a profile loss (at zero incidence) and shock loss components and that the two are additive.

Expression for the shock loss coefficient, K_s

The head loss, h, in a turbomachine blade-to-blade passage is related [2] to the stagnation pressure loss, ΔP_0, as follows:

$$h = \frac{\Delta P_0}{\rho g} \tag{5}$$

where ρ is the fluid density. The expression linking the drag coefficient, C_D, to the stagnation pressure loss in a cascade [2] was then reformulated as follows:

$$W_m = \sqrt{\frac{(s/c)\Delta P_0 \cos\beta_m}{1/2\rho C_D}} \tag{6}$$

where s/c is the solidity, β_m is the mean relative flow angle, which is measured relative to the radial direction, and W_m is the mean relative flow velocity, which is also given by:

$$W_m = \frac{V_{rm}}{\cos \beta_m} = \frac{Q}{\pi t D_m \cos \beta_m} = \frac{2Q}{\pi t(D_1 + D_2)\cos \beta_m} \tag{7}$$

where t is the axial thickness of the impeller, D the diameter and V_r the radial velocity.

The case of a simple impeller of constant axial width is considered. Hence equations 6 and 7 give:

$$\frac{\sqrt{2}\sqrt{(c/s)}Q}{\pi t(D_1 + D_2)} = \sqrt{\frac{\cos^3 \beta_m}{C_D}}\sqrt{\frac{\Delta P_0}{\rho}} \tag{8}$$

Using equation 8, the following expression for the shock loss is obtained:

$$h_s = \frac{2(c/s)(Q - Q_0)^2}{g[\pi t(D_1 + D_2)]^2} \frac{1}{\left(\sqrt{\frac{\cos^3 \beta_m}{C_D}} - \sqrt{\frac{\cos^3 \beta_{m0}}{C_{D0}} \frac{\Delta P_{00}}{\Delta P_0}}\right)} \tag{9}$$

Since the shock loss coefficient is also defined by $h_s = K_s (Q - Q_0)^2$, the following expression for the shock loss coefficient, K_s, is obtained:

$$K_s = \frac{2(c/s)\sec^3 \beta_m}{g[\pi t(D_1 + D_2)]^2} \frac{1}{\left(1 - \sqrt{\frac{\cos^3 \beta_{m0}}{\cos^3 \beta_m} \frac{C_D}{C_{D0}} \frac{\Delta P_{00}}{\Delta P_0}}\right)^2} \tag{10}$$

where β_{mo}, the mean vane angle, is the average of the inlet and outlet vane angles.

The parameters in equation 10 need to be defined so that the average value of the shock loss coefficient can be calculated. The following simplifications are proposed.

Stagnation pressure loss

The following approximation for the ratio of stagnation pressure loss is used:

$$\frac{\Delta P_{00}}{\Delta P_0} = \frac{h_{f\,\min}}{h_f} = \frac{1 - \eta_{\max}}{1 - \eta_{av}} \tag{11}$$

where h_f is the head loss due to friction and η the efficiency.

Mean blade angle, β_m

For a centrifugal pump impeller, β_m is give by:

$$\tan \beta_m = \frac{\tan \beta_1 + \tan \beta_2 \left(\frac{R_1}{R_2}\right)}{1 + \frac{R_1}{R_2}} \tag{12}$$

For blades in the form of a logarithmic spiral, it will easily be seen that the mean blade angle is the same as the inlet or outlet blade angle.

The flow angles, β_1 and β_2

These are defined by:

$$\beta_1 = \beta_1' - i$$
$$\beta_2 = \beta_2' - \delta \tag{13}$$

where the angles of incidence and deviation, i and δ, respectively, are assumed to have the same order of magnitude for normal operation. Then, for a simple impeller with vanes in the form of a logarithmic spiral of vane angle 67°, and on the assumption that the maximum pump flow rate is equal to twice the nominal flow rate, say, the maximum and minimum incidences, i_{mas} and i_{min}, are:

$$i_{\max} = 23\,°$$
$$i_{\min} = -19\,° \quad (14)$$

The above then yields an average incidence, i_{av}, of 2°, which, we assume, has the same order of magnitude as the average deflection, δ_{av}.

Average efficiency
The average efficiency is obtained on the assumption that the variation of the hydraulic efficiency with flow rate approximates a sine function. The average value can then be computed as follows:

$$\eta_{av} = \frac{\eta_{\max}}{\pi}\int_0^{\pi} \sin \Phi \mathrm{d}\Phi \approx \frac{2}{\pi}\eta_{\max} \quad (15)$$

where an acceptable value of $\eta_{\max}$, the maximum efficiency, is 85%.

Frictional loss coefficient

Useful parameters

Length of spine of an impeller passage
If D_1 and D_2 are the internal and external diameters of an impeller and β the blade angle measured with respect to the radial direction, the length of the spine of an impeller passage with blades in the form of a logarithmic spiral is given by:

$$L = \frac{D_2 - D_1}{2\cos\beta} \quad (16)$$

Mean passage width
The mean width, D_{m}, of an impeller with Z vanes is given by:

$$D_m = \frac{\pi}{2Z}(D_1 + D_2) \quad (17)$$

Passage hydraulic diameter
If t is the axial width of an impeller passage, then its hydraulic diameter, D_{h}, is given by:

$$D_h = \frac{2tD_m}{t + D_m} \quad (18)$$

Expansion angle of an equivalent flat diffuser
The expansion angle, α, of an equivalent flat diffuser is given by:

$$\tan\frac{\alpha}{2} = \frac{\pi}{2ZL}(D_2 - D_1) \quad (19)$$

The average diameter, D_{av}, of curvature

It is the arithmetic average of the diameters of curvature at inlet and outlet of an impeller. It is defined by [4]:

$$D_{av} = \frac{2}{k_{av}} \tag{20}$$

where k_{av}, the average curvature, can be shown to be given by the following expression:

$$k_{av} = \frac{2 \sin \beta}{D_2 - D_1} \ln\left(\frac{D_2}{D_1}\right) \tag{21}$$

Flow rate, Q

For the simple impeller considered, the flow rate, Q, is given by:

$$Q = \frac{\pi^2 D_1^2 N t \cot \beta_1}{60} \tag{22}$$

where N is the rotational speed in rpm. Hence, Q_0, the nominal flow rate, can be calculated.

Expression for head loss coefficient

For a pump impeller passage modelled as the combination of a diffuser and a duct of the shape of a spiral, the expression for the head loss, H_f, in the passage is:

$$H_f = \frac{K_d W_1^2}{2g} + \frac{K_{sp} W_m^2}{2g} \tag{23}$$

Then using the well known geometries of the inlet and outlet velocity triangles, we obtain the following expression for the head loss coefficient:

$$K_f = \frac{K_d}{2g(\pi D_1 t \cos \beta_1)^2} + \frac{K_{sp}(D_1 + D_2)^2}{8g(\pi D_1 D_2 t \cos \beta_m)^2} \tag{24}$$

Frictional loss coefficients for a diffuser (K_d) and a spiral (K_{sp})

For the smooth spiral and rough diffuser model used, the two components for the frictional loss coefficient are obtained using correlations. The loss coefficient, K_{sp}, for the spiral is given by [4]:

$$K_{sp} = A_1 B_1 C_1 + \frac{\lambda L}{D_h} \tag{25}$$

where λ is the coefficient of friction of a smooth surface. The following correlation for the friction coefficient, λ, was used [4]:

$$\lambda = 0.079\,\mathrm{Re}^{-0.25} + 0.1025\left(\frac{D_h}{D_{av}}\right)^{0.9} \tag{26}$$

where Re is the Reynolds number. A_1, B_1, C_1 are constants whose values depend on the angle of the elbow, θ, the ratios R_{av}/D_h and t/D_h. On the other hand, the diffuser head loss coefficient, K_d, is given by [3]:

$$K_d = 3.2 \tan\frac{\alpha}{2}\sqrt[4]{\tan\frac{\alpha}{2}}\left(1-\frac{D_1}{D_2}\right)^2 + \frac{\lambda_d}{4\sin\frac{\alpha}{2}}\left\{\frac{t}{D_h}\left(1-\frac{D_1}{D_2}\right)+0.5\left[1-\left(\frac{D_1}{D_2}\right)^2\right]\right\} \quad (27)$$

Results

Calculations were done for a simple impeller with vanes of negligible thickness. Table 1 gives the geometry of the impeller and other parameters used in the calculation and Table 2 the results. The constants A, B and C are given with equation 2 in the zone of the characteristics without recirculation and by equation 4 in the zone with recirculation. All the calculations were done using the drag coefficients for the NACA 23012 profile, for which aerofoil data are available. Fig. 1 gives the characteristics obtained. It is noted that the present procedure predicts the head loss at the closed valve reasonably well [3], that is,

$$H_{cv} \approx 0.6\frac{U_2^2}{2g}$$

We then attempted to predict the head flow characteristic of an available centrifugal pump (a PEDROLLO NF130A). The nominal power and impeller diameter are 2.2 kW and 12.4 cm respectively. Table 3 gives the geometry of the impeller and other parameters and Table 4 the parameters used for obtaining the characteristics.

Fig. 2 shows the theoretical, predicted and manufacturer's characteristics. We note that the agreement between the theoretical and the manufacturer's characteristics is again good with the valve closed (zero flow rate). In the region of maximum pump

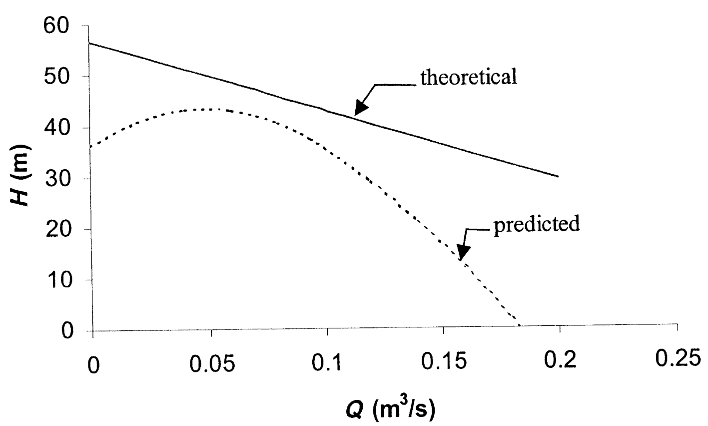

Fig. 1 *Predicted and theoretical characteristics of the pump geometry chosen.*

TABLE 1 *Geometry of the impeller and parameters used in the calculations*

D_1 (m)	D_2 (m)	D_{av} (m)	D_h (m)	L (m)	Z	α (degree)	β_i (degree)	i_{av} (degree)	ω (degree)	C_{Dmin}	C_{Dmax}
0.150	0.300	0.235	0.059	0.177	6	23.5	67	2	97	0.075	0.180

TABLE 2 *Calculated values*

A_1	B_1	C_1	λ	K_{sp}	K_d	K_f	$(K_s)_{av}$	Q (m^3/s)	Q_0 (m^3/s)	H_{th} (m)	H_s (m)	H_f (m)	A	B	C	H_R (m)	η
1.024	1.024	1.115	0.030	1.269	0.187	751.773	824.200	0.103	0.094	42.443	0.071	8.034	Using equation 2				0.807
													49.294	18.256	−1575.9	34.338	
													Using equation 4				
													35.981	301.21	−3079.5	34.208	

TABLE 3 *Geometry of the impeller and parameters of the PEDROLLO NF130A pump used in the calculations*

D_1 (m)	D_2 (m)	D_{av} (m)	D_h (m)	L (m)	Z	α (degree)	β_i (degree)	i_{av} (degree)	ω (degree)	C_{Dmin}	C_{Dmax}
0.0671	0.124	0.099	0.031	0.076	6	23.5	68	2	97	0.075	0.180

TABLE 4 *Calculated values, corresponding to the PEDROLLO NF130A pump*

A_1	B_1	C_1	λ	K_{sp}	K_d	K_f	$(K_s)_{av}$	Q (m^3/s)	Q_D (m^3/s)	H_{th} (m)	H_s (m)	H_f (m)	A	B	C	H_R (m)	η
0.966	0.966	1.098	0.067	1.191	0.200	12927.9	16470.4	0.021	0.019	25.585	0.065	5.984	Using equation 2				0.6017
													29.888	151.346	−38230.5	15.4465	
													Using equation 4				
													20.031	1161.01	−64086.4	15.3439	

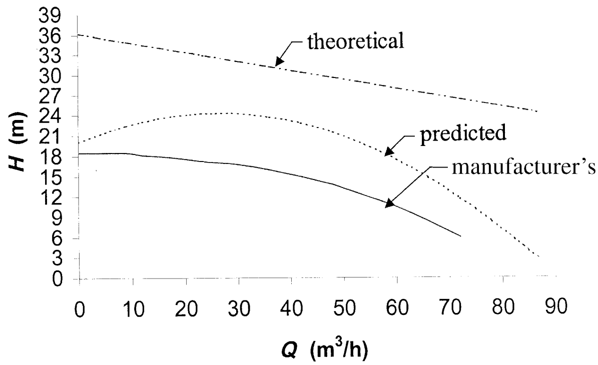

Fig. 2 *Real, theoretical and manufacturer's characteristics of the PEDROLLO NF130A pump.*

efficiency, the theoretical characteristic lies midway between the ideal and that of the manufacturer. It is thought that this discrepancy is due to:

(1) scale effects (because of the very small size of the pump impeller);
(2) use of aerodynamic characteristics of the vanes which may be inaccurate;
(3) the fact that the present model has been developed for a centrifugal pump with purely radial vanes;
(4) volute and entrance losses, which are not considered.

Conclusion

The basis for teaching centrifugal pump characteristics using new procedures for taking care of shock losses and losses due to recirculation produced realistic results. The procedure predicts the head at the closed valve reasonably well. This is hardly surprising because of the dominance of centrifugal effects in the impeller at zero flow rate. We therefore expect the procedure to predict the characteristics of a purely radial impeller reasonably well.

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