Node-voltage method using 'virtual current sources' technique for special cases

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Abstract A new technique based on the use of 'virtual current sources' makes any planar or nonplanar electric circuit solvable using the node-voltage method. The node-voltage method is systematized and it is shown how difficulties encountered until now with special circuit categories can be overcome.

Keywords convertible voltage source; node-voltage method; non-convertible voltage source; special cases; virtual current source

DC circuit analysis by the node-voltage method may be applied to circuits in which most of their sources are current sources, or to circuits for which the mesh-current method cannot be applied (i.e. nonplanar circuits). Many introductory electric circuit textbooks[1–9] utilize this method, which is based on a systematic application of Kirchhoff's current law. The node-voltage method provides a general procedure for analysing planar or nonplanar circuits using node voltages as the circuit variables; it provides a simple and systemic method for circuits that contain only independent current sources.

Difficulties with this method (from a systematic standardization and pedagogical effectiveness point of view) occur when the circuit also contains dependent sources and when there are voltage sources (independent or dependent) that are not transformable to current sources (independent or dependent) respectively. The problem of non-convertible voltage sources has been tackled in the past by using supernodes, something that students are not able to easily understand and apply; specifically, generalization and standardization of the problem to special cases in electric circuits has not been easy for them.

These difficulties are removed by the method presented by Gottling,[10] who shows how to write node analysis matrix equations for a linear circuit by inspection and derives a general matrix solution for the node-voltage vector. This matrix solution has a form similar to Wilson's matrix solution[11] for operational amplifier circuits, but it is more general as it includes all four types of dependent source, op amps, and mutually coupled inductances.

This paper, in addition to presenting the node-voltage method in a systematic way, solves the problem of non-convertible voltage sources by introducing the concept of 'virtual current sources',[3] which, as the Gottling method, also overcomes the above mentioned limitations.

The term virtual current source means that: a non-convertible voltage source (independent or dependent) is substituted by a current source (independent or dependent respectively), that has a value equal to the current through this voltage source and which is unknown.

The node-voltage method

Analysing planar or nonplanar circuits in a systematic and standard way using the node-voltage method depends on the kind of sources that exist in the circuit and also on whether the existing voltage sources are convertible or non-convertible. Based on these considerations, the node-voltage method can be examined for four different cases of planar or nonplanar circuits of n nodes.

(a) Planar or nonplanar circuit with independent (current or/and voltage) sources but, with all possibly existing voltage sources convertible.

In such a case, the voltage sources are initially transformed to current sources and then in the resulting equivalent circuit the following steps are applied:

Step 1: One node of the circuit is defined as the reference node. Despite the fact that this node can be freely chosen, the aim is to choose the node to which most branches are connected, since in this case the resulting equations are simplified.

Step 2: For the remaining $h = n - 1$ nodes, after numbering them, the node voltages with respect to the reference node are defined.

Step 3: The node equations are written in matrix form as follows:

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1h} \\ G_{21} & G_{22} & \cdots & G_{2h} \\ \vdots & \vdots & \vdots & \vdots \\ G_{h1} & G_{h2} & \cdots & G_{hh} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_h \end{bmatrix} = \begin{bmatrix} \Sigma i_1 \\ \Sigma i_2 \\ \vdots \\ \Sigma i_h \end{bmatrix}$$

where: G_{ii}, $\forall i = 1, 2, 3, \ldots, h$ denotes the *self-conductance* of the $(\text{node})_i$ and is equal to the sum of all conductances at this node.

$G_{ij} = G_{ji}$, $\forall i \neq j$, $i, j = 1, 2, 3, \ldots, h$ denotes the *mutual conductance* of $(\text{node})_i$ and $(\text{node})_j$, and is equal to the sum of the conductances directly connecting these nodes. Its sign is always (–).

Σi_j, $\forall_j = 1, 2, 3, \ldots, h$ is the algebraic sum of the values of all current sources connected to $(\text{node})_j$. The values of those sources whose current flows towards the node are taken positive while, in the opposite case, they are taken as negative.

Step 4: The resulting $h \times h$ linear system is solved using the Cramer method or the matrix inversion method and the node voltages $v_1, v_2, v_3, \ldots, v_h$ are thus known.

Step 5: The voltages of all branches are calculated combining the node voltages and as a consequence the currents of all circuit elements are known.

In other words, the solution of the electric circuit is complete.

Notes

- The conductance matrix is symmetrical since $G_{ij} = G_{ji}$, $\forall i \neq j$.
- If the circuit contains a current source in series with a resistance, then this resistance is eliminated during the procedure for finding the node voltages or a node (with the appropriate numbering) is considered between the source and the resistance. When the resistance is eliminated, one must be careful to consider this resistance in the final calculation of the dissipated power, otherwise the power balance will not be valid. To avoid mistakes, it is recommended considering a node between the source and the resistance.

Example a

For the circuit of Fig. 1, calculate the currents i_x, i_y and the voltage v_x using the node-voltage method. Next, show that the power developed is equal to the power dissipated.

Solution: The reference node is defined and the remaining nodes are numbered (Fig. 2). Then, the corresponding node voltages are also defined.

The node equations in matrix form are:

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \Sigma i_1 \\ \Sigma i_2 \\ \Sigma i_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.05 & -0.025 & 0 \\ -0.025 & 0.075 & -0.05 \\ 0 & -0.05 & 0.173 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -7.5 \\ 12.5 \\ -8.333 \end{bmatrix}$$

$$\begin{aligned} v_1 &= -84\text{ V} \\ \Rightarrow v_2 &= 132\text{ V} \\ v_3 &= -10\text{ V} \end{aligned}$$

Hence,

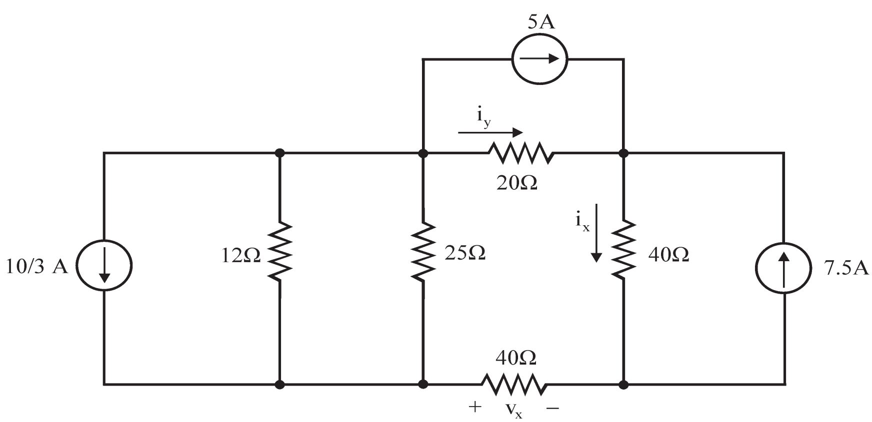

Fig. 1 *Circuit for case a.*

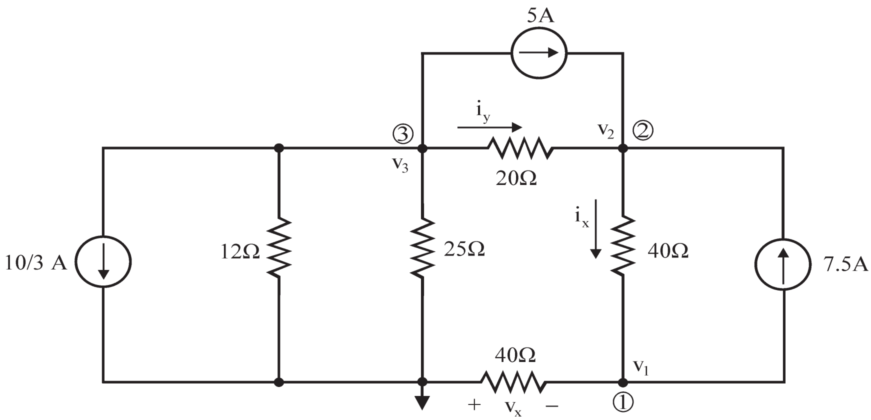

Fig. 2 *Equivalent circuit of Fig. 1.*

$$i_x = \frac{v_2 - v_1}{40} = 5.4\,\text{A} \qquad i_y = \frac{v_3 - v_2}{20} = -7.1\,\text{A} \qquad v_x = 0 - v_1 = 84\,\text{V}$$

For the power balance:

$$p_{(5\text{A})} = -(v_2 - v_3)\times 5 = -710\,\text{W} \qquad p_{(7.5\text{A})} = -(v_2 - v_1)\times 7.5 = -1620\,\text{W}$$

$$p_{\left(\frac{10}{3}\text{A}\right)} = v_3 \cdot \frac{10}{3} = -33.333\,\text{W}$$

$$p_{\text{RESIST.}} = \frac{v_3^2}{12} + \frac{v_3^2}{25} + \frac{v_1^2}{40} + \frac{(v_3 - v_2)^2}{20} + \frac{(v_2 - v_1)^2}{40} = 2363.333\,\text{W}$$

Thus

$$\left.\begin{aligned} p_{\text{DELIV.}} = p_{(5\text{A})} + p_{(7.5\text{A})} + p_{\left(\frac{10}{3}\text{A}\right)} &= 2363.333\,\text{W} \\ p_{\text{DISSIP.}} = p_{\text{RESIST.}} &= 2363.333\,\text{W} \end{aligned}\right\} \Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

(b) Planar or nonplanar circuit, with independent (current or/and voltage) sources but, with at least a voltage source not transformable to current source or for which the transformation is difficult (*special case*).

In such a case, the following steps are executed:

Step 1: In situations where voltage sources are non-convertible, *virtual current sources* are considered, with values equal to the corresponding current values flowing through the non-convertible voltage sources of the given circuit.

Step 2: One node of the circuit is defined as the reference node as in the previous case.

Step 3: The remaining $h = n - 1$ nodes are numbered and the node voltages are defined with respect to the reference node.

Step 4: The node equations are written in matrix form as in the previous case. However, in this case *virtual* current sources are taken together with the non-virtual sources, and included in the terms Σi_j.

Step 5: For each *virtual* current source, an equation is introduced in the matrix that describes the corresponding non-convertible voltage source with a linear combination of the unknown node voltages of the problem, eliminating each time an equation that contains a *virtual* current. The remaining equations needed for the solution are taken from the initial form of the matrix, as they are (those that do not contain unknown voltages other than the node voltage) or as they result after the appropriate additions or subtractions in order to eliminate the *virtual* currents appearing initially.

By doing so, the new matrix equation no longer represents Ohm's law, but is simply an algebraically $h \times h$ equivalent system, which can lead to determinations of the node voltages.

Step 6: The resulting $h \times h$ linear system is solved as in the previous case and so the node voltages are readily available.

Step 7: The voltages of all branches are calculated combining the node voltages and as a consequence the currents of all circuit elements are known, except those flowing through the non-convertible voltage sources (that is the virtual currents). The calculation of these currents is done using the equations that were eliminated from the initial matrix form of the node equations, since the node voltages are already known.

In other words, the solution of the electric circuit is completed.

Notes

- A non-convertible voltage source is a source that has no resistance in series with it.
- If there is a current source in series with a resistance, the problem is treated as in the previous case.

Example b

For the circuit of Fig. 3, using the node-voltage method show that the power developed is equal to the power dissipated.

Solution: The voltage source of 8 V is transformed to a current source of 2 A in parallel with a resistance of 4 Ω. The voltage source of 10 V is non-convertible, since there is no resistance in series with it. Therefore, a virtual current source is considered at its location (replacing it) with a value i_x equal to the current flowing through the 10 V voltage source of the given circuit (Fig. 4). Next, defining the reference node and taking into account that the current source of 4 A is in series with the resistance of 8 Ω (hence a node is positioned and numbered between them), all nodes are numbered and the corresponding node voltages defined.

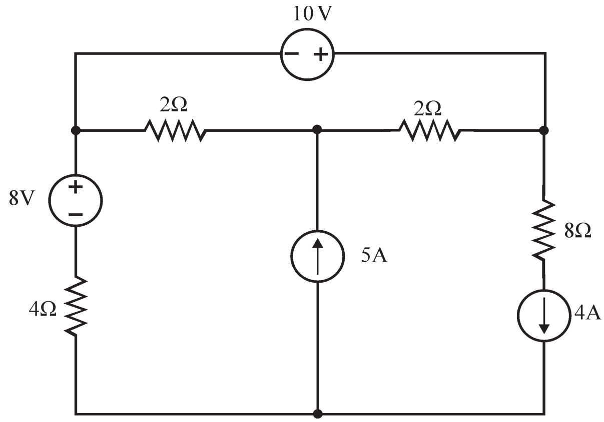

Fig. 3 *Circuit for case b.*

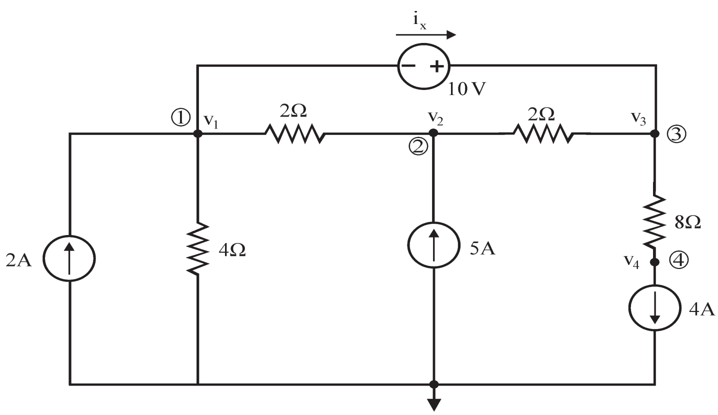

Fig. 4 *Equivalent circuit of Fig. 3.*

Based on the above, for the circuit of Fig. 4, the node equations in matrix form are:

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \Sigma i_1 \\ \Sigma i_2 \\ \Sigma i_3 \\ \Sigma i_4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.75 & -0.5 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & -0.5 & 0.625 & -0.125 \\ 0 & 0 & -0.125 & 0.125 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 2 - i_x \\ 5 \\ i_x \\ -4 \end{bmatrix} \quad (1)$$

Substituting the first line of relationship (1) by the equation $v_3 - v_1 = 10\,\text{V}$, which applies to the voltage source of 10 V, leaving the second line as is, and substituting the third line by the fourth line and the fourth line by the sum of the first and third lines, the following algebraically equivalent system results:

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ -0.5 & 1 & -0.5 & 0 \\ 0 & 0 & -0.125 & 0.125 \\ 0.75 & -1 & 0.625 & -0.125 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ -4 \\ 2 \end{bmatrix} \Rightarrow \begin{matrix} v_1 = 12\,\text{V} \\ v_2 = 22\,\text{V} \\ v_3 = 22\,\text{V} \\ v_4 = -10\,\text{V} \end{matrix}$$

In order to find the current flowing through the non-convertible voltage source, the following is considered:

The third line of relationship (1) gives: $i_x = -0.5v_2 + 0.625v_3 - 0.125v_4 = 4\text{A}$
Hence,

$$p_{(10\text{V})} = -10 \cdot i_x = -40\,\text{W} \qquad p_{(5\text{A})} = -v_2 \cdot 5 = -110\,\text{W}$$

$$p_{(4\text{A})} = v_4 \cdot 4 = -40\,\text{W} \qquad p_{(8\text{V})} = \frac{v_1 - 8}{4} \cdot 8 = 8\,\text{W}$$

$$p_{\text{RESIST.}} = \frac{(v_1 - v_2)^2}{2} + \frac{(v_2 - v_3)^2}{2} + \frac{(v_3 - v_4)^2}{8} + \frac{(v_1 - 8)^2}{4} = 182\,\text{W}$$

Therefore,

$$\left.\begin{matrix} p_{\text{DELIV.}} = p_{(10\text{V})} + p_{(5\text{A})} + p_{(4\text{A})} = 190\,\text{W} \\ p_{\text{DISSIP.}} = p_{\text{RESIST.}} + p_{(8\text{V})} = 190\,\text{W} \end{matrix}\right\} \Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

(c) Planar or nonplanar circuit, with independent and dependent (current or/and voltage) sources but, with all voltage sources that possibly exist in the circuit convertible.

In this case, the voltage sources (independent, dependent) are initially transformed to current sources (independent, dependent respectively), and then in the resulting equivalent circuit the following steps are applied:

Step 1: One node of the circuit is defined as the reference node as in case (a).
Step 2: The remaining $h = n - 1$ nodes are numbered and the node voltages are defined with respect to the reference node.
Step 3: The node equations are written in matrix form as in case (a).
Step 4: The dependent quantities appearing in the matrix are expressed with respect to the unknown node voltages. However, this implies that the unknown node voltages appear in the second part of the matrix form of the equations as well.
Step 5: The elements of the equation are rearranged (when needed) so that the unknown node voltages appear only on the left of the equations.
Step 6: The resulting $h \times h$ linear system is solved as in case (a), and so the node voltages are readily available.

Step 7: The voltages of all branches are calculated combining the node voltages and as a consequence the currents of all circuit elements are known.

In other words, the solution of the electric circuit is completed.

Notes

- A dependent voltage source is considered convertible when there is a resistance in series with it and simultaneously the dependent quantity of this or any other dependent source is not located at this series resistance. If something like this were to happen, source transformation would result in the elimination of the dependent quantity and therefore further steps for the problem solution would be difficult or impossible.
- An independent voltage source is considered convertible when there is a resistance in series with it and simultaneously the dependent quantity of a dependent current or voltage source does not appear at this resistance or at the source (for the same reason as previously detailed).
- If there is a current source in series with a resistance, the problem is treated as in the previous cases.

Example c

For the circuit of Fig. 5, using the node-voltage method show that the power developed is equal to the power dissipated.

Solution: Initially, the dependent voltage source is transformed into its corresponding dependent current source. Then, after defining the reference node and after numbering the remaining nodes, the circuit takes the form of Fig. 6.

The node equations in matrix form are:

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \Sigma i_1 \\ \Sigma i_2 \\ \Sigma i_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.25 & -0.2 & 0 \\ -0.2 & 0.325 & -0.1 \\ 0 & -0.1 & 0.55 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5i_0 \\ 0 \\ 2.3i_0 + 24 \end{bmatrix}$$

$$= \begin{bmatrix} 5\dfrac{v_2}{40} \\ 0 \\ 2.3\dfrac{v_2}{40} + 24 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.25 & -0.325 & 0 \\ -0.2 & 0.325 & -0.1 \\ 0 & -0.1575 & 0.55 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} \Rightarrow \begin{matrix} v_1 = 156\,\text{V} \\ v_2 = 120\,\text{V} \\ v_3 = 78\,\text{V} \end{matrix}$$

Hence,

$$p_{(96\text{V})} = -96 \cdot \frac{96 - v_3}{4} = -432\,\text{W} \quad p_{(5i_0)} = -5i_0 \cdot v_1 = -5\frac{v_2}{40} \cdot v_1 = -2340\,\text{W}$$

$$p_{(11.5i_0)} = \frac{v_3 - 11.5i_0}{5} \cdot 11.5i_0 = 300.15\,\text{W}$$

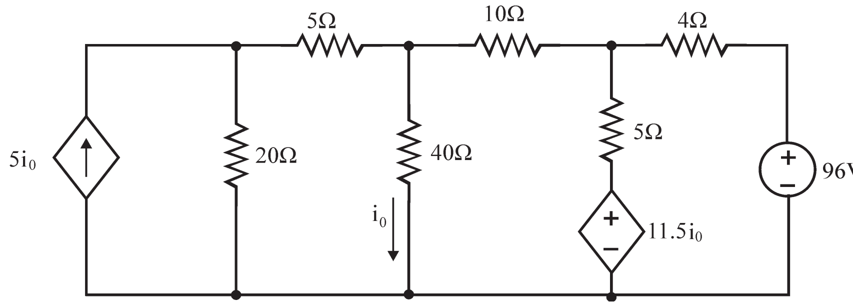

Fig. 5 *Circuit for case c.*

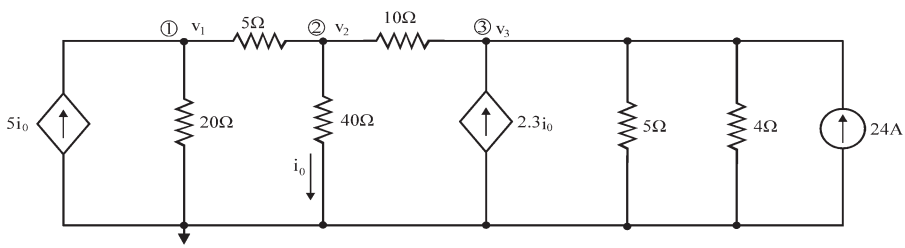

Fig. 6 *Equivalent circuit of Fig. 5.*

$$p_{\text{RESIST.}} = \frac{v_1^2}{20} + \frac{(v_1 - v_2)^2}{5} + \frac{v_2^2}{40} + \frac{(v_2 - v_3)^2}{10} + \frac{(96 - v_3)^2}{4} + \frac{(v_3 - 11.5i_0)^2}{5}$$
$$= 2471.85\,\text{W}$$

Thus,

$$\left.\begin{aligned} p_{\text{DELIV.}} &= p_{(96\text{V})} + p_{(5i_0)} = 2772\,\text{W} \\ p_{\text{DISSIP.}} &= p_{\text{REIST.}} + p_{(11.5i_0)} = 2772\,\text{W} \end{aligned}\right\} \Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

(d) Planar or nonplanar circuit with independent and dependent (current or/and voltage) sources but, with at least a voltage source (independent or dependent) not transformable to current source (independent or dependent respectively) or for which the transformation is difficult (special case).

In such a case, the following steps are applied:

Step 1: For voltage sources that are non-convertible, *virtual current sources* are considered with values equal to the corresponding current values flowing through the non-convertible voltage sources of the given circuit.
Step 2: One node of the circuit is defined as the reference node as in case (a).
Step 3: The remaining $h = n - 1$ nodes are numbered and the node voltages are defined with respect to the reference node.

Step 4: The node equations are written in matrix form as in case (a). However, in this case the *virtual* current sources together with the existing (non-virtual) sources are included in the terms Σi_j.

Step 5: For each *virtual* current source, an equation is introduced in the matrix that describes the corresponding non-convertible voltage source with a linear combination of the unknown node voltages of the problem, eliminating each time an equation that contains a *virtual* current. The remaining equations needed for the solution are taken from the initial form of the matrix, as they are (those that do not contain unknown voltages other than the node voltage) or as they result after the appropriate additions or subtractions made in order to eliminate the *virtual* currents appearing initially.

By doing so, the new matrix form of the equations no longer represents Ohm's law, but it is simply an algebraically $h \times h$ equivalent system, which can lead to determination of the node voltages.

Step 6: The dependent quantities appearing in the matrix form are expressed with respect to the unknown node voltages. However, this implies that the unknown node voltages appear in the second part of the matrix form of the equations.

Step 7: The elements of the equations are rearranged (when needed) so that the unknown node voltages appear only on the left of the equations.

Step 8: The resulting $h \times h$ linear system is solved as in case (a) and so the node voltages are readily available.

Step 9: The voltages of all branches are calculated combining the node voltages and as a consequence the currents of all circuit elements are known, except those flowing through the non-convertible voltage sources (that is the virtual currents). The calculation of these currents is done using the equations that were eliminated from the initial matrix form of the node equations, since the node voltages are already known.

In other words, the solution of the electric circuit is completed.

Notes

- A dependent voltage source is considered non-convertible when there is no resistance in series with it and when there is, the dependent quantity of this or any other dependent source is located at this series resistance. If source transformation were to happen, it would result in the elimination of the dependent quantity and therefore further steps for the problem solution would be difficult or impossible.
- An independent voltage source is considered non-convertible when there is no resistance in series with it or when there is, the dependent quantity of a dependent current or voltage source does not appear at this resistance or at the source (for the same reason as previously detailed).
- If there is a current source in series with a resistance, the problem is treated as in the previous case.

Example d

For the circuit of Fig. 7, using the node-voltage method show that the power developed is equal to the power dissipated.

Solution: Since the dependent voltage source of $0.5v_\varphi$ is non-convertible (there is no resistance in series with it), a *virtual current source* is considered in its position (replacing it) with a value i_y equal to the current flowing through this voltage source of the given circuit (Fig. 8). The independent voltage source of 10 V, despite the fact that there is a resistance of 20 Ω in series with it, is considered non-convertible, since the dependent quantity of the $0.5v_\varphi$ voltage source is located at this resistance. Therefore, a *virtual current source* with a value of i_x is considered, located at the 10 V source. This value is equal to the current flowing through this source of the given circuit. A correctly numbered node is placed between this current source and the resistance 20 Ω.

A properly numbered node is also placed between the independent current source of 1A and the resistance of 10 Ω. Defining the reference node and numbering the remaining nodes, the circuit takes the form of Fig. 8.

The node equations in matrix form are:

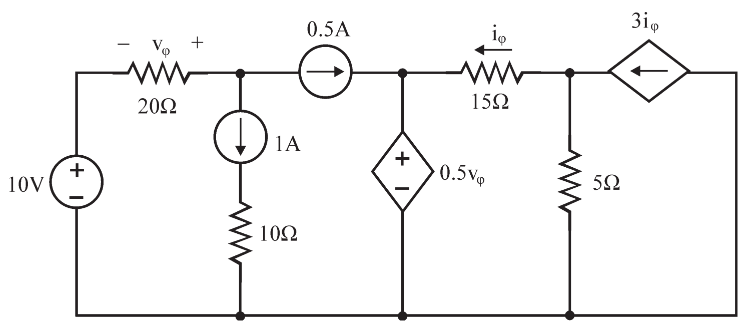

Fig. 7 *Circuit for case d.*

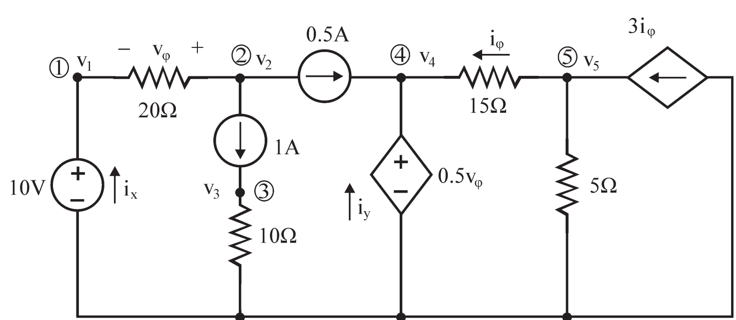

Fig. 8 *Equivalent circuit of Fig. 7.*

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} \\ G_{41} & G_{42} & G_{43} & G_{44} & G_{45} \\ G_{51} & G_{52} & G_{53} & G_{54} & G_{55} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} \Sigma i_1 \\ \Sigma i_2 \\ \Sigma i_3 \\ \Sigma i_4 \\ \Sigma i_5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.05 & -0.05 & 0 & 0 & 0 \\ -0.05 & 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.067 & -0.067 \\ 0 & 0 & 0 & -0.067 & 0.267 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} i_x \\ -1.5 \\ 1 \\ 0.5 + i_y \\ 3i_\varphi \end{bmatrix} \tag{2}$$

Substituting the first line of relationship (2) by the equation $v_1 = 10\,V$ dealing with the independent 10 V source, the second line by the equation $v_4 = 0.5v_\varphi$ dealing with the $0.5v_\varphi$ dependent voltage source, the third line by the second line, the fourth line by the third line and leaving the fifth line as is, the following algebraically equivalent circuit is derived:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0.05 & 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & -0.067 & 0.267 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0.5v_\varphi \\ -1.5 \\ 1 \\ 3i_\varphi \end{bmatrix} = \begin{bmatrix} 10 \\ 0.5(v_2 - v_1) \\ -1.5 \\ 1 \\ 3\dfrac{v_5 - v_4}{15} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.5 & -0.5 & 0 & 1 & 0 \\ -0.05 & 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.133 & 0.067 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -1.5 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} v_1 = 10\,V \\ v_2 = -20\,V \\ v_3 = 10\,V \\ v_4 = -15\,V \\ v_5 = 30\,V \end{matrix}$$

In order to find the currents flowing through the non-convertible voltage sources, the following are considered:

From the first line of relationship (2): $i_x = 0.05v_1 - 0.05v_2 \Rightarrow i_x = 1.5A$
From the fourth line of relationship (2): $0.5 + i_y = 0.067v_4 - 0.067v_5 \Rightarrow i_y = -3.5A$

Hence,

$$p_{(0.5A)} = 0.5 \cdot (v_2 - v_4) = -2.5\,W \quad p_{(1A)} = 1 \cdot (v_2 - v_3) = -30\,W$$

$$p_{(10V)} = -10 \cdot i_x = -15\,W \quad p_{(0.5v_\varphi)} = -0.5v_\varphi \cdot i_y = -0.5(v_2 - v_1) \cdot i_y = -52.5\,W$$

$$p_{(3i_\varphi)} = -3i_\varphi \cdot v_5 = -3\frac{v_5 - v_4}{15} \cdot v_5 = -270\,W$$

$$p_{\text{RESIST.}} = \frac{(v_2 - v_1)^2}{20} + \frac{v_3^2}{10} + \frac{(v_5 - v_4)^2}{15} + \frac{v_5^2}{5} = 370\,\text{W}$$

Thus,

$$\left.\begin{array}{r} p_{\text{DELIV.}} = p_{(0.5\text{A})} + p_{(1\text{A})} + p_{(10\text{V})} + p_{(0.5\text{v}_\varphi)} + p_{(3\text{i}_\varphi)} = 370\,\text{W} \\ p_{\text{DISSIP.}} = p_{\text{RESIST.}} = 370\,\text{W} \end{array}\right\} \Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

Conclusions

The classification of planar or nonplanar electric circuits into four categories, as were examined in this paper, enables the student to solve any circuit following similar procedures.

With respect to the special cases (b), (d) dealt with *virtual current sources*, this results in the non-differentiation of these cases regarding the overall methodological steps to be followed. This is because the concept of a node is not modified, as done when supernodes are used, but the student sees the nodes from the beginning, without having to search for an appropriate approach to solving the problem.

Another equally important advantage of the use of *virtual current sources* is the immediate determination of the currents flowing through the non-convertible voltage sources, given that the node voltages are known, since their currents are already expressed in the equations written in matrix form. So, the power developed by these sources is easy to calculate and therefore the proof of the power balance does not present any difficulties.

Special attention must be paid to the conditions under which a voltage source is transformed to a current source; this is because whereas a voltage source is convertible when there is a resistance in series with it, for methodological purposes it should be considered non-convertible when the conditions mentioned in case (d) are not met. Also, one must be careful, when a current source is in series with a resistance, to consider a node between them, as noted earlier.

Finally, the node-voltage method, as has been analysed, can obviously be used for planar or nonplanar circuits in sinusoidal steady state (AC circuits). However, a necessary condition is that all circuit sources are of the same frequency (otherwise the principle of superposition is used). If all sources are of the same frequency, the node-voltage method starts after the circuit transformation to the frequency domain.

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