Use of random variables in power transmission engineering

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Abstract The use of random variables and the application of probabilistic techniques for power system analysis has become an important issue due to the increased uncertainties faced by modern power systems. In the techno-economic analysis of an installation, especially at the design stage, several discrete values have to be considered for each variable, such as voltage, power, power angle, etc. Therefore, in handling such problems, it is advisable to use random variable ranges as explained in the paper taking the area of power transmission engineering as an example.

Keywords mean; power flow; random variables; standard deviation; transmission line

Introduction

Most phenomena in nature, society and technology are subject to random variations. This randomness is often ignored when considering such phenomena. To determine the performance of a system, the nature of the relationship between output variable and input variables is interpreted by using the mean value of the output variable instead of the extreme value. In the past, this has generally been taken into account by multiplying the known mean values by safety factors.

A rather better approach to the above problem is to replace the deterministic determination of the average trends by a thorough statistical treatment of the stochastic or random phenomena. This approach can also be seen in power system analysis. A variety of power system analyses is essential to operate an electric power system and to plan its expansion properly. Common types of power system studies are short-circuit analysis, power flow analysis, stability analysis, and analysis of electromagnetic transients.

A rigorous assessment of the risk of insulation breakdown caused by steep switching surges requires that both the switching surge magnitudes and the insulation withstand strength be recognized as random variables and described in terms of their probability distributions. The random nature of faults and contingency situations need to be accounted for in short-circuit studies. The major concern of statistical techniques in the area of transient stability is with the random aspects of fault type, fault location, fault clearing phenomenon, and operating conditions, which affect stability. In power flow studies, there is uncertainty associated with demand forecast, and the degree of uncertainty is quantified by the forecast variance. The uncertainty in forecast demands results in uncertainty in power flows as well as voltage levels.

Several probabilistic power flow formulations have been published and applied to the problem described in this paper since the 1970s.[1–3] In these formulations, loads are typically the inputs and are considered the given random variables, and the random nature of the voltage magnitudes and angles is ignored. This paper presents a few examples for evaluating the mean and standard deviation of power flow through a transmission line, illustrating the use of random voltage magnitudes at the line ends and the random power angle due to uncertain forecast demands. A transmission line interconnecting two power systems with random operational inputs as given below is considered.

Case1: The sending-end and receiving-end voltages are the random inputs. The power angle is assumed to remain constant. It is required to determine the mean and standard deviation of the active power flow through the transmission line, assuming normal distribution.

Case 2: Voltage magnitudes at the line ends are assumed to remain equal and constant. The power angle is a random variable distributed uniformly within a certain interval. The power flow is the output random variable.

Case 3: The power angle is a random variable uniformly distributed within a certain interval. The magnitude of the sending end voltage is held constant. The receiving end voltage is a random number normally distributed with known values of mean and standard deviation. It does not depend on the sending-end voltage magnitude and the power angle. The power flow is the output random variable.

In this paper, detailed derivations are given of the means and standard deviations of the output variable, namely the power flow in a long transmission line, in terms of the means and standard deviations of the input variables for the three cases considered. The work is based on Ref. 4, where the above three cases are considered for the simple short-line case wherein line resistance and line capacitance are ignored. The study reported in this paper will aid students in learning the application of random variables to power transmission engineering. It will also assist students and instructors in the comprehensive application of random variables to other areas of power engineering.

Power flow through a transmission line

The sending end and receiving end voltages of a transmission line are Vs and Vr respectively. The generalized circuit constants of the line are $\mathbf{A} = |A|\angle\alpha$ and $\mathbf{B} = |B|\angle\beta$ respectively, and the load angle is δ. Note that bold letters indicate complex quantities.

The real and reactive powers P and Q at the receiving end are:

$$P = \frac{Vs \cdot V\mathrm{r}\cos(\beta - \delta)}{|B|} - \frac{|A|}{|B|} \cdot V\mathrm{r}^2 \cos(\beta - \alpha) \tag{1}$$

$$Q = \frac{V\mathrm{s} \cdot V\mathrm{r}\sin(\beta - \delta)}{|B|} - \frac{|A|}{|B|} \cdot V\mathrm{r}^2 \sin(\beta - \alpha) \tag{2}$$

As mentioned earlier, Ref. 4 ignores line resistance and capacitance and considers only the line reactance X, and thus the results use the simple power flow formula:

$$P = \frac{Vs \cdot Vr \sin(\delta)}{X} \tag{3}$$

Properties of random variables

If X is a random variable with probability density function $p(x)$, and $Y = f(x)$, then the mean of Y will be given by

$$M(Y) = M[f(x)] = \int_{-\infty}^{\infty} f(x) p(x) \mathrm{d}x \tag{4}$$

The variance is given by

$$D(Y) = D[f(x)] = \int_{-\infty}^{\infty} [f(x)]^2 p(x) \mathrm{d}x - [M(Y)]^2 \tag{5}$$

The following rules apply to calculations using means and variances of independent random variables X_1, X_2:

$$M(X_1 + X_2) = M(X_1) + M(X_2) \tag{6}$$

$$M(X_1, X_2) = M(X_1)M(X_2) \tag{7}$$

$$M(aX + b) = a \cdot M(X) + b \tag{8}$$

$$D(X_1 + X_2) = D(X_1) + D(X_2) \tag{9}$$

Note that $D(X_1 - X_2)$ is also equal to $D(X_1) + D(X_2)$.

$$D(X_1, X_2) = D(X_1)D(X_2) + [M(X_1)]^2 D(X_2) + [M(X_2)]^2 D(X_1) \tag{10}$$

$$D(aX + b) = a^2 D(X) \tag{11}$$

Properties for the three cases studied

Vs, Vr random inputs and δ constant

Mean values $M(Vs)$, $M(Vr)$ and corresponding standard deviations σ_{Vs}, σ_{Vr} are known. Thus $D(Vs) = \sigma_{Vs}^2$ and $D(Vr) = \sigma_{Vr}^2$.

Mean value of the power transmitted is given by

$$M(P) = \frac{M(Vs) \cdot M(Vr)}{|B|} \cos(\beta - \delta) - \frac{|A|}{|B|} \cdot [M(Vr)]^2 \cos(\beta - \alpha) \tag{12}$$

Variance of the power transmitted is given by

$$D(P) = D\left[\frac{Vs \cdot Vr\cos(\beta - \delta)}{|B|} - \frac{|A|}{|B|} \cdot Vr^2 \cos(\beta - \alpha)\right]$$
$$= \frac{\cos^2(\beta - \delta)}{|B|^2}\left\{D(Vs) \cdot D(Vr) + [M(Vs)]^2 D(Vr) + [M(Vr)]^2 D(Vs)\right\}$$
$$- \frac{|A|^2}{|B|^2}\cos^2(\beta - \alpha)\left\{[D(Vr)]^2 + 2D(Vr)[M(Vr)]^2\right\} \tag{13}$$

using equations (9) and (10).

Vs, *Vr* constant, δ a random input

δ is a random variable with uniform distribution within the range δ_1 and δ_2. Let $C = \delta_2 - \delta_1$. Then the probability density function for uniform distribution is $p(\delta) = 1/C$.

Mean value of the power transmitted is given by

$$M(P) = \frac{Vs \cdot VrM[\cos(\beta - \delta)]}{|B|} - \frac{|A|}{|B|} \cdot Vr^2 \cos(\beta - \alpha) \tag{14}$$

where

$$M[\cos(\beta - \delta)] = \int_{\delta_1}^{\delta_2} \cos(\beta - \delta) \cdot p(\delta) d\delta$$
$$= \frac{[\sin(\beta - \delta_1) - \sin(\beta - \delta_2)]}{C} \qquad \text{using equation (4) and (6)}$$

Variance of the power transmitted is given by

$$D(P) = D\frac{[VsVr\cos(\beta - \delta)]}{|B|}$$
$$= \frac{(VsVr)^2}{|B|^2} D[\cos(\beta - \delta)] \qquad \text{using equation (11)} \tag{15}$$

where

$$D[\cos(\beta - \delta)]$$
$$= \frac{(VsVr)^2}{|B|^2}\left\{\int_{\delta_1}^{\delta_2} \cos^2(\beta - \delta) \cdot p(\delta) d\delta - [M(\cos(\beta - \delta))]^2\right\} \qquad \text{using equation (5)}$$
$$= \frac{(VsVr)^2}{|B|^2}\left\{0.5 + \frac{[\sin(2(\beta - \delta_1)) - \sin(2(\beta - \delta_2))]}{4C} - \frac{[\sin(\beta - \delta_2) - \sin(\beta - \delta_1)]^2}{C^2}\right\}$$

Vs is constant, *Vr* and δ are random inputs

δ is a random variable with uniform distribution within the range δ_1 and δ_2. $C = \delta_2 - \delta_1$ and $p(\delta) = 1/C$.

Vr does not depend on Vs and δ and is a normally distributed random variable. Mean value $M(V\mathrm{r})$ and corresponding standard deviation $\sigma_{V\mathrm{r}}$ are known. Thus, $M(V\mathrm{r}) = \sigma_{V\mathrm{r}}^2$.

Using equations (6) and (7), the mean value of the power transmitted is given by

$$\begin{aligned} M(P) &= (V\mathrm{s}/|B|)M(V\mathrm{r})\cdot M(\cos(\beta-\delta)) - (|A|/|B|)\cos(\beta-\alpha)\{(M(V\mathrm{r}))^2\}, \\ &= (V\mathrm{s}/|B|)M(V\mathrm{r})\cdot\frac{[\sin(\beta-\delta_1)-\sin(\beta-\delta_2)]}{C} \\ &\quad - (|A|/|B|)\cos(\beta-\alpha)\{(M(V\mathrm{r}))^2\} \end{aligned} \tag{16}$$

Using equation (6), the variance of the power transmitted is given by

$$\begin{aligned} D(P) &= D\frac{[V\mathrm{s}V\mathrm{r}\cos(\beta-\delta)]}{|B|} - D\left[\frac{|A|}{|B|}\cdot V\mathrm{r}^2\cos(\beta-\alpha)\right] \\ &= (V\mathrm{s}/|B|)^2\cdot D\{V\mathrm{r}\cdot\cos(\beta-\delta)\} - [|A|\cos(\beta-\alpha)/|B|]^2\cdot D(V\mathrm{r}^2) \\ &= (V\mathrm{s}/|B|)^2\Big\{D(V\mathrm{r})D(\cos(\beta-\delta)) + D(V\mathrm{r})[M(\cos(\beta-\delta))]^2 \\ &\quad + D(\cos(\beta-\delta))[M(V\mathrm{r})]^2\Big\} - [(|A|/|B|)\cos(\beta-\alpha)]^2\{[D(V\mathrm{r})]^2 + 2[M(V\mathrm{r})]^2\} \end{aligned} \tag{17}$$

where

$$D(\cos(\beta-\delta)) = 0.5 + \frac{\sin(2(\beta-\delta_1)) - \sin(2(\beta-\delta_2))}{4C} - \{M(\cos(\beta-\delta))\}^2$$

and

$$M(\cos(\beta-\delta)) = \frac{[\sin(\beta-\delta_1)-\sin(\beta-\delta_2)]}{C}$$

Using equations (6) and (7), the mean value of the reactive power transmitted is given by

$$M(Q) = (V\mathrm{s}/|B|)M(V\mathrm{r})\cdot M(\sin(\beta-\delta)) - (|A|/|B|)\sin(\beta-\alpha)\{(M(V\mathrm{r}))^2\} \tag{18}$$

where

$$M(\sin(\beta-\delta)) = \frac{[\cos(\beta-\delta_2)-\cos(\beta-\delta_1)]}{C}$$

Using equation (6), the variance of the reactive power transmitted is given by

$$D(Q) = D\frac{[Vs Vr\sin(\beta-\delta)]}{|B|} - D\left[\frac{|A|}{|B|}\cdot Vr^2\sin(\beta-\alpha)\right]$$
$$= (Vs/|B|)^2\cdot D\{Vr\cdot\sin(\beta-\delta)\} - [|A|\sin(\beta-\alpha)/|B|]^2\cdot D(Vr^2)$$
$$= (Vs/|B|)^2\left\{D(Vr)D(\sin(\beta-\delta)) + D(Vr)[M(\sin(\beta-\delta))]^2\right.$$
$$\left. + D(\sin(\beta-\delta))[M(Vr)]^2\right\} - [(|A|/|B|)\sin(\beta-\alpha)]^2\{[D(Vr)]^2 + 2[M(Vr)]^2\} \tag{19}$$

where

$$D(\sin(\beta-\delta)) = 0.5 + \frac{\sin(2(\beta-\delta_2)) - \sin(2(\beta-\delta_1))}{4C} - \{M(\sin(\beta-\delta))\}^2$$

and

$$M(\sin(\beta-\delta)) = \frac{[\cos(\beta-\delta_2) - \cos(\beta-\delta_1)]}{C}$$

Equations (9) and (10) are used to derive equation (19).

Numerical results

Transmission line data

220 kV, 50 Hz transmission line, 300 km long
A = 0.948∠ 0.0121
B = 126.13∠1.3555 ohms

Case studies

Table 1 gives the means and standard deviations of the power flows for the three cases. Additional information pertaining to the input data for the three cases is given below.

Case1: The standard deviations of the sending-end and receiving-end voltages are σ_{Vs} and σ_{Vr} respectively. The power angle has a constant value of 30°. The value of the active power, which will exceed a certain value with the probability of 0.0668, is also calculated and given in Table 1. The active power so calculated is assumed to follow a normal distribution. The cumulative distribution of a normally distributed random variable involves integration of the normal distribution analytically. Consequently, the normal distribution with mean M and variance σ^2 is transformed to the unit normal distribution with mean 0 and variance 1. The cumulative distribution of the unit normal distribution is available in tabular form for easy reference in most books[5–7] on probability and statistics. In case 1, for the given value of $p = 0.0668$, $1 - p = 0.9332$. From the table[6] of cumulative normal distribution function, a value y corresponding to the value of $1 - p$ is read out. The power P with

TABLE 1 *Mean and standard deviation of power flow for the stochastic input data of Cases 1–3*

Case 1 Input data		Case 2 Input data		Case 3 Input data	
$M(Vs)$	219 kV	Vs	220 kV	Vs	230 kV
$M(Vr)$	212 kV	Vr	220 kV	$M(Vr)$	215 kV
σ_{Vs}	7 kV	δ_1	–30°	σ_{Vr}	5 kV
σ_{Vr}	6 kV	δ_2	60°	δ_1	30°
δ	30°			δ_2	45°
Output data		Output data		Output data	
$M(P)$	171.74 MW	$M(P)$	76.63 MW	$M(P)$	220.43 MW
σ_P	10.14 MW	σ_P	146.12 MW	σ_P	20.31 MW
Value of P with probability 0.0668	186.95 MW	–	–	$M(Q)$	–86.34 MVAr
–	–	–	–	σ_Q	25.88 MVAr

the probability p is then obtained from $P = y^*\sigma_P + M(P)$, where $M(P)$ and σ_P are respectively the mean and standard deviation of P.

Case 2: The angle δ is a random variable uniformly distributed within the interval $[\delta_1, \delta_2]$ and the end voltages are maintained constant.

Case3: The angle δ is a random variable uniformly distributed within the interval $[\delta_1, \delta_2]$. The receiving-end voltage Vr does not depend on Vs and δ and is a normally distributed random variable having a standard deviation σ_{Vr}. In this case, the mean and standard deviation of the reactive power are also calculated.

The deterministic values of power flow for the nominal input data of Cases 1–3 are also calculated from equations (1) and (2) and given in Table 2. These deterministic values of power flow are compared below with the mean values of the probabilistic power flows.

In case 1, the mean value of the power flow at a constant load angle of 30° and for the assumed random values of voltage magnitude at nominal values of voltage is 171.74 MW. This is equal to the deterministic mean value that corresponds to the nominal values of angle and voltage magnitude respectively. The standard deviation of the power flow has a small value of 10.14 MW because of the assumed random variations in the voltage magnitude.

In case 2, where the load angle is assumed to have a uniform distribution between –30° and 60°, the mean value of power flow is 76.63 MW, which is approximately equal to the deterministic value corresponding to constant voltage magnitude and a median load angle of 15°. The standard deviation in this case, however, has a high value of 146.12 MW because of the assumed large interval of 90° for the uniform distribution of load angle.

TABLE 2 *Deterministic mean values of power flow for the nominal input data of Cases 1–3*

Case 1 Input data		Case 2 Input data		Case 3 Input data	
Vs	219 kV	Vs	220 kV	Vs	230 kV
Vr	212 kV	Vr	220 kV	Vr	215 kV
δ	30°	δ	15°	δ	37.5°
Output data		Output data		Output data	
Mean value of power flow	171.74 MW	Mean value of power flow	94.19 MW	Mean value of power flow	217.59 MW
				Mean value of reactive power flow	−101.54 MVAr

In case 3, where the load angle is assumed to vary between 30° and 45°, the mean value of power flow is 220.43 MW, which is approximately equal to the deterministic mean value for the nominal voltage magnitude and for a median load angle value of 37.5°. The standard deviation in this case is small because of the small interval of 15° in the assumed uniform distribution for the load angle.

Conclusions

Although a transmission line is designed to carry a particular amount of power, in actual practice the line may be forced to carry a higher power during the integrated operation of the power system. Hence, it becomes necessary to consider the probabilistic aspects of transmission line operation at the design stage. This study will help students to understand the applications of random variables in the area of power transmission engineering. Detailed calculation steps are given for each case. This will assist students and instructors in carrying out a comprehensive study of the application of random variables to more complicated and important problems in power engineering.

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