Relationship between harmonics and symmetrical components

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Abstract New terminology is introduced to make clear the relationship between harmonics and symmetrical components. Three-phase sets are classified in terms of symmetrical sets and asymmetrical sets. Subclasses are introduced with the names symmetrical balanced sets, symmetrical unbalanced sets, asymmetrical balanced sets and asymmetrical unbalanced sets to show that a three-phase set can resolve to either one, two or three symmetrical component sets. The results from four case studies show that these subclasses and their resolution to symmetrical component sets improve understanding of harmonic analysis of systems having balanced and unbalanced harmonic sources and loads.

Keywords asymmetrical sets; harmonic flows; harmonic sources; symmetrical component sets; symmetrical sets

Any periodic wave shape can be broken down into or analysed as a fundamental wave and a series of harmonics.[1]

Three-phase harmonic analysis requires a clear understanding of the relationship between symmetrical component injections from harmonic sources (e.g. adjustable speed drives, ASDs) and their relationship to harmonic flows (symmetrical components) arising from the application of a harmonic source to a linear system.

A limited number of references contain brief information concerning harmonics and symmetrical components.[1–5] Reference 1, provides a paragraph on this topic and uses the heading 'Relationship between Harmonics and Symmetrical Components'. It includes a table that is supported by a brief explanatory paragraph. The table expresses harmonics in terms of positive, negative and zero sequences. It states that these sequences are for harmonics in balanced three-phase systems. The heading refers to symmetrical components while the content refers to balanced three-phase systems. Herein lies the anomaly. Classically, symmetrical components (especially zero sequence) are only applied in unbalanced systems. The following questions arose after reading the Ref. 1 paragraph.

(a) Do symmetrical components (especially zero sequence), in the classical sense, apply in balanced as well as unbalanced non-sinusoidal systems and is this a break from tradition?
(b) What do the terms, symmetrical, asymmetrical, balanced, unbalanced and symmetrical components mean?
(c) What are the conditions under which a system must operate so that harmonics resolve to positive, negative and zero sequences and is the table given in Ref. 1 correct?

The terminology used is found inadequate for describing non-sinusoidal systems. There is thus a need to introduce a three-phase terminology that will show the relationship and make the comparison between injections (currents) and harmonic flows (voltages and currents) meaningful.

References 3 provides the basis for the solution by providing definitions for 'three-phase sets', 'symmetrical sets' and 'symmetrical component sets'.

The purpose of this paper is to introduce an approach to harmonic analysis based on the classification of three-phase sets and to make to comparison between injections from harmonic sources and corresponding harmonic flows quantifiable by expressing the results in terms of the number of symmetrical component sets found.

Harmonic flows and their resolution to symmetrical components depends upon the magnitudes and phase sequences of the injections from a harmonic source, on the system's sequence impedances, on three- and four-wire connections and on whether the customer's linear load on the system is balanced or unbalanced. Therefore, what is injected in terms of symmetrical component sets by a harmonic source is not necessarily received by the system, i.e. the harmonic flows may resolve to one, two or three symmetrical component sets and this depends upon the type of three-phase set found. Therefore, any three-phase harmonic may be partially made up of any of the symmetrical component sets.[4,6]

Four case studies are reported and they show a novel method for teaching the flow of power system harmonics. It is important to use case studies as part of one's teaching as they link learning to concepts and improve understanding. They show how the method of symmetrical components can be extended to a system's response to harmonic flows. When taught as a group, the four case studies improve cognitive skills by showing that the symmetrical component responses under unbalanced situations are different to the balanced state.

Classification of three-phase sets

At a point/node within a sinusoidal system, a set of currents (or voltages) is found, for example:

$$i_a = \sqrt{2} I_{\mathrm{A}} \cos(\omega t + \alpha_{\mathrm{a1}}) \tag{1}$$

$$i_b = \sqrt{2} I_{\mathrm{B}} \cos(\omega t + \alpha_{\mathrm{b1}}) \tag{2}$$

$$i_c = \sqrt{2} I_{\mathrm{C}} \cos(\omega t + \alpha_{\mathrm{c1}}) \tag{3}$$

Such a set is called a three-phase set (TPS) and is defined as 'a group of three interrelated currents (or voltages) that have the same period'.[3]

Therefore, a system is comprised of a number of TPSs. The type of TPS found at the various points/nodes in a given system depends upon the system configuration.

Symmetrical and asymmetrical sets

The magnitudes and phase displacement of the three phasors of a TPS are variables. A TPS can therefore be either a symmetrical set (SS) or an asymmetrical set (AS).

If each phasor of a TPS is equal in magnitude and has either a 120° or 0° phase displacement, the set is a SS.[3,7] A TPS which does not have the characteristics of a

SS in an AS. A TPS can have either an *abc*, *acb* or an in-phase order of rotation for its phasors. Figure 1 shows the classification of a TPS in the form of a diagram.

There are thus six classes of TPS.

Meaning of the terms balanced and unbalanced

If the three phasors (I_A, I_B and I_C) of a TPS at a given point in a system have the same magnitude and are displaced from each other by 120°, the set is balanced. In this case, the phasors form an equilateral triangle and their sum is zero. With reference to a TPS at a given point in a system, the term balanced means the phasors of the set sum to zero ($I_A + I_B + I_C = 0$). Therefore, if the three phasors of a set at a given point in a system do not sum to zero ($I_A + I_B + I_C \neq 0$), the TPS is unbalanced.[8–10]

Symmetrical and asymmetrical balanced and unbalanced sets

If the TPS is a SS with 120° phase displacement, the phasors sum to zero, therefore the TPS is a symmetrical balanced set (SBS). There are thus two types of SBS, one with an *abc* order and one with and *acb* order. Whereas, if the TPS is a SS with a 0° phase displacement, the phasors do not sum to zero and the TPS is a symmetrical unbalanced set (SUS).

If the phasors of an AS sum to zero, the TPS is an asymmetrical balanced set (ABS). Whereas, if the phasor sum of an AS is not zero, the TPS is an asymmetrical unbalanced set (AUS).[9]

Figure 2 shows the subclasses of TPSs.

Symmetrical component sets resolved from symmetrical and asymmetrical sets

Understanding the relationship between symmetrical components and three-phase harmonics provides a firm foundation for power system analysis.[2]

The method of symmetrical components as an analytical tool is traditionally used to examine sinusoidal systems under balanced and unbalanced customer loading conditions.[11] As harmonics are sinusoidal quantities, the method can be extended to steady-state harmonic analysis.[4]

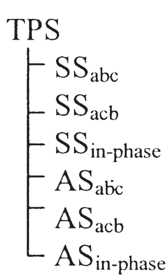

Fig. 1 *Classification of a three-phase set (TPS).*

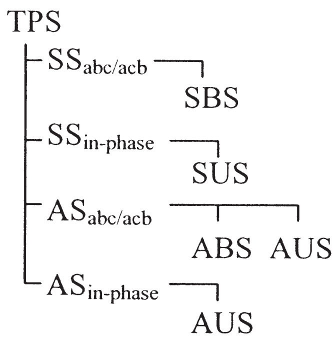

Fig. 2 *Subclasses of three-phase sets.*

The symmetrical components of any TPS ($I_A + I_B + I_C$) are:

$$I_0 = 1/3\left(I_A + I_B + I_C\right) \tag{4}$$

$$I_1 = 1/3\left(I_A + aI_B + a^2 I_C\right) \tag{5}$$

$$I_2 = 1/3\left(I_A + a^2 I_B + aI_C\right) \tag{6}$$

The symmetrical component transformation is a general mathematical technique developed by Fortescue and provides that any TPS may be resolved into symmetrical components.[11]

Therefore, if the TPS in equation (4) is:

(i) SBS or SUS

The TPS resolves to only one symmetrical component set (SCS). The other two SCSs are absent.

If the order of the SBS is *abc*, it resolves to a positive sequence symmetrical component set (SCS^+), whereas if it is *acb*, it resolves to a negative sequence symmetrical component set (SCS^-).

If the set is SUS, it resolves to a zero sequence symmetrical component set (SCS^z).[6,12]

(ii) ABS or AUS

The TPS resolves to more than one SCS.

If the set is ABS, it resolves to two SCSs; one SCS^+ and one SCS^-.

If the set is AUS, it resolves to three SCSs; a SCS^+, SCS^- and SCS^Z.[9]

Figure 3 and Table 1 show the SCSs resolved from the subclasses of TPSs.

Modelling three-phase harmonic current sources in terms of symmetrical component sets

Harmonic sources are considered to be injection sources and for most studies are treated as simple sources of harmonic currents.[4]

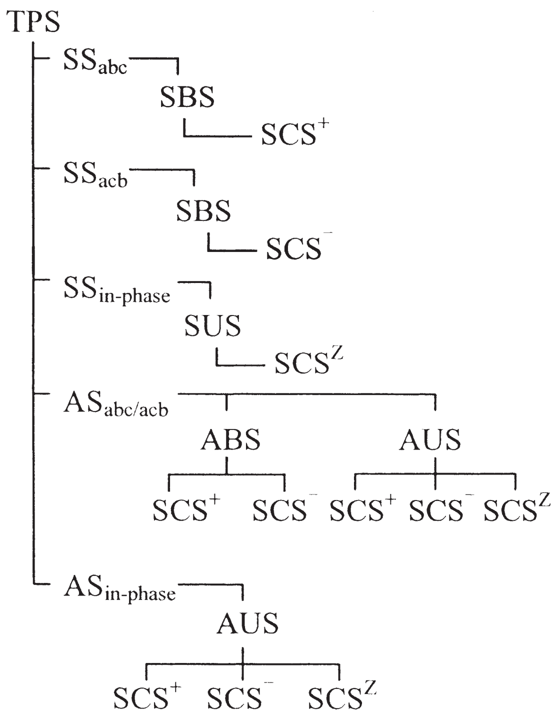

Fig. 3 *SCSs resolved from TPSs.*

TABLE 1 *SCSs resolved from subclasses of TPSs*

Three-phase sets (TPS)		Symmetrical component sets (SCS)		
Type	Order	Positive	Negative	zero
SBS	*abc*	SCS^{+}	Absent	Absent
SBS	*acb*	Absent	SCS^{-}	Absent
SUS	in-phase	Absent	Absent	SCS^{Z}
ABS	*abc/acb*	SCS^{+}	SCS^{-}	Absent
AUS	*abc/acb*	SCS^{+}	SCS^{-}	SCS^{Z}
	in-phase	SCS^{+}	SCS^{-}	SCS^{Z}

Balanced source

The main contributor to system harmonic distortion is the six-pulse convertor. Under perfect conditions it is an ideal convertor and its wave shapes are periodic. The Fourier series for the current in its 'A' phase is:[11]

$$i_A = (2/\pi)\sqrt{3}I_d\big[(\cos\omega t) - (1/5)(\cos 5\omega t) + (1/7)(\cos 7\omega t) - (1/11)(\cos 11\omega t) + (1/13)(\cos 13\omega t) - (1/17)(\cos 17\omega t) + (1/19)(\cos 19\omega t) - \cdots\big] \quad (7)$$

where $(2/\pi)\sqrt{3}\ I_d$ is the maximum value of the fundamental frequency component (I_{1m}).

The RMS magnitude of the fundamental frequency and each harmonic component are $I_1 = [(2/\pi)\ \sqrt{3}\ I_d] \div \sqrt{2}$ and $I_h = I_1/h$, respectively, where h is the harmonic number. Triplen harmonic components are absent. Only $6k \pm 1$ harmonics are present, for integer values $k = 1, 2, 3, \ldots, n$. They are called characteristic harmonics.[13]

The general expression for the currents in the A, B and C phases are therefore:

$$\begin{aligned} i_A &= I_{1m}\cos\omega t - I_{5m}\cos 5\omega t + I_{7m}\cos 7\omega t - I_{11m}\cos 11\omega t \\ &\quad + I_{13m}\cos 13\omega t - I_{17m}\cos 17\omega t + I_{19m}\cos 19\omega t \cdots \end{aligned} \tag{8}$$

$$\begin{aligned} i_B &= I_{1m}\cos(\omega t - 120°) - I_{5m}\cos(5\omega t - 240°) + I_{7m}\cos(7\omega t - 120°) \\ &\quad - I_{11m}\cos(11\omega t - 240°) + I_{13m}\cos(13\omega t - 120°) \\ &\quad - I_{17m}\cos(17\omega t - 240°) + I_{19m}\cos(19\omega t - 120°) \cdots \end{aligned} \tag{9}$$

$$\begin{aligned} i_C &= I_{1m}\cos(\omega t + 120°) - I_{5m}\cos(5\omega t + 240°) + I_{7m}\cos(7\omega t + 120°) \\ &\quad - I_{11m}\cos(11\omega t + 240°) + I_{13m}\cos(13\omega t + 120°) \\ &\quad - I_{17m}\cos(17\omega t + 240°) + I_{19m}\cos(19\omega t + 120°) \cdots \end{aligned} \tag{10}$$

Changing − to + polarities, the equations are:[14,15]

$$\begin{aligned} i_A &= I_{1m}\cos\omega t + I_{5m}\cos(5\omega t + 180°) + I_{7m}\cos 7\omega t + I_{11m}\cos(11\omega t + 180°) \\ &\quad + I_{13m}\cos 13\omega t + I_{17m}\cos(17\omega t + 180°) + I_{19m}\cos 19\omega t \cdots \end{aligned} \tag{11}$$

$$\begin{aligned} i_B &= I_{1m}\cos(\omega t - 120°) + I_{5m}\cos(5\omega t - 60°) + I_{7m}\cos(7\omega t - 120°) \\ &\quad + I_{11m}\cos(11\omega t - 60°) + I_{13m}\cos(13\omega t - 120°) \\ &\quad + I_{17m}\cos(17\omega t - 60°) + I_{19m}\cos(19\omega t - 120°) \cdots \end{aligned} \tag{12}$$

$$\begin{aligned} i_C &= I_{1m}\cos(\omega t + 120°) + I_{5m}\cos(5\omega t + 60°) + I_{7m}\cos(7\omega t + 120°) \\ &\quad + I_{11m}\cos(11\omega t + 60°) + I_{13m}\cos(13\omega t + 120°) \\ &\quad + I_{17m}\cos(17\omega t + 60°) + I_{19m}\cos(19\omega t + 120°) \cdots \end{aligned} \tag{13}$$

At each frequency a SBS is obtained. By definition, harmonics result from periodic steady-state operating conditions and therefore their prediction should be formulated in terms of harmonic phasors, i.e. in the frequency domain,[11] namely:

ωt	$5\omega t$	$7\omega t$
$I_{A1}/\_0°$	$I_{A5}/\_+180°$	$I_{A7}/\_0°$
$I_{B1}/\_-120°$	$I_{B5}/\_-60°$	$I_{B7}/\_-120°$
$I_{C1}/\_+120°$	$I_{C5}/\_+60°$	$I_{C7}/\_120°$

Spectrum
- $TPS_{(\omega t)}$ - SS_{abc} - SBS - SCS^{+}
- $TPS_{(5\omega t)}$ - SS_{acb} - SBS - SCS^{-}
- $TPS_{(7\omega t)}$ - SS_{abc} - SBS - SCS^{+}
- $TPS_{(11\omega t)}$ - SS_{acb} - SBS - SCS^{-}
- $TPS_{(13\omega t)}$ - SS_{abc} - SBS - SCS^{+}
- etc.

Fig. 4 *SCSs used to model an ideal six-pulse convertor.*

$11\omega t$	$13\omega t$	$17\omega t$	$19\omega t$
$I_{A11}/\_+180°$	$I_{A13}/\_0°$	$I_{A17}/\_+180°$	$I_{A19}/\_0°$
$I_{B11}/\_-60°$	$I_{B13}/\_-120°$	$I_{B17}/\_-60°$	$I_{B19}/\_-120°)$
$I_{C11}/\_+60°$	$I_{C13}/\_+120°$	$I_{C17}/\_+60°$	$I_{C19}/\_+120°)$

The SBSs at 1^{st}, 7^{th}, 13^{th} and 19th harmonic frequencies have an *abc* order of rotation, equal magnitudes and are 120° apart. They are thus SCS^{+}s. At the 15th, 11^{th} and 17^{th} harmonic frequencies the SCSs have an *acb* order of rotation, equal magnitudes and are 120° apart and are thus SCS^{-}s. SCSs resolved from SBSs can thus be used to model an ideal six-pulse convertor, as shown in Fig. 4.

Each injection is only one SCS, they, however, have different frequencies. As all the injections are SBSs the convertor can be said to operate as a balanced harmonic current source.[16]

Unbalanced source

Normally the convertor injects only characteristic harmonics. If not fired symmetrically the convertor injects non-characteristic harmonics (triplens and even harmonics) besides characteristic harmonics.

Since convertors typically inject currents with half-wave symmetry, even harmonics are ignored. Under such non-ideal conditions, the actual magnitudes of the TPSs of a six-pulse convertor are not inversely proportional to the harmonic number h, therefore

$$I_h \neq I_1/h \tag{14}$$

Magnitudes are based on measurements or published data.

TPSs at triplen harmonic frequencies typically have an in-phase order of rotation and are SUSs. Their injections are therefore treated as SCS^{Z}s.

Under non-ideal conditions the convertor spectrum would be[1] as shown in Fig. 5.

As the source injects both SUSs and SBSs it can be said to operate as an unbalanced harmonic source[16] (Table 2).

Spectrum

- $TPS_{(\omega t)}$ - SS_{abc} - SBS - SCS^{+}
- $TPS_{(3\omega t)}$ - $SS_{\text{in-phase}}$ - SUS - SCS^{Z}
- $TPS_{(5\omega t)}$ - SS_{acb} - SBS - SCS^{-}
- $TPS_{(7\omega t)}$ - SS_{abc} - SBS - SCS^{+}
- $TPS_{(9\omega t)}$ - $SS_{\text{in-phase}}$ - SUS - SCS^{Z}
- $TPS_{(11\omega t)}$ - SS_{acb} - SBS - SCS^{-}
- $TPS_{(13\omega t)}$ - SS_{abc} - SBS - SCS^{+}
- etc.

Fig. 5 *Convertor spectrum under non-ideal conditions.*

TABLE 2 *Source operating as an unbalanced harmonic source*

Unbalanced harmonic source			
NR	Symmetrical components		
h	Sequence	*h*	Sequence
1	Positive*	11	Negative*
3	Zero	13	Positive*
5	Negative*	15	Zero
7	Positive*	17	Negative*
9	Zero	etc	

* Indicates a balanced harmonic source.

Case studies

A three-phase four-wire radial distribution system is used for the four case studies conducted.

The system is comprised of a three-phase sinusoidal voltage source (star point connected to neutral), a distribution network (HV and LV sections including a star/star transformer, both sides have neutral returns providing four-wire distribution) and a load (customer).

The customer load is comprised of a motor load (three-wire), a linear star load with a neutral return, a single-phase linear load with a neutral return and a harmonic current source (six-pulse convertor).

The results of the case studies are given in Table 3.

The SUPERHARM software program is utilized as it is able to analyse both balanced and unbalanced non-sinusoidal systems.[15]

The injections from the harmonic source are compared to corresponding harmonic flows in the various sections of the system in terms of SSs, ASs and SCSs. The number of SCSs is shown in brackets in Table 3.

TABLE 3 *Results of case studies*

		Injections from a harmonic source compared to harmonic flows						
			Distribution network			Induction motor load		
		Harmonic current source (injections)	HV and LV sections					
			L–N(V)	L–L(V)	(A)	L–L(V)	(A)	
Case	Harmonic frequency		Type of set (number of symmetrical component sets in brackets)					Customer linear load
1	Fundamental	SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	Balanced
2		SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	
3		SBS(1)	AUS(3)	ABS(2)	(AUS)(3)	ABS(2)	ABS(2)	Unbalanced
4		SBS(1)	AUS(3)	ABS(2)	AUS(3)	ABS(2)	ABS(2)	
1	$6k \pm 1$	SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	Balanced
2		SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	SBS(1)	
3		SBS(1)	AUS(3)	ABS(2)	AUS(3)	ABS(2)	ABS(2)	Unbalanced
4		SBS(1)	AUS(3)	ABS(2)	AUS(3)	ABS(2)	ABS(2)	
1	Triplen	None	N/A	N/A	N/A	N/A	N/A	Balanced
2		SUS(1)	SUS(1)	None	SUS(1)	None	None	
3		None	N/A	N/A	N/A	N/A	N/A	Unbalanced
4		SUS(1)	AUS(3)	ABS(2)	AUS(3)	ABS(2)	ABS(2)	

The harmonic current source for cases 1 and 3 is modelled as a balanced source, whereas for cases 2 and 4 an unbalanced source is used.

The same spectrum for the harmonic source is used for all cases, except triplen harmonics are added for the unbalanced source investigations.

For cases 1 and 2 the customer load on the system is balanced (the single-phase linear load is ignored) whereas for cases 3 and 4 the load is unbalanced.

In case 1, the fundamental and $6k \pm 1$ injections and corresponding harmonic flows are all single SCSs. Therefore, the SCS^+s and SCS^-s that are injected, propagate through the three- and four-wire sections of the system. This is expected as the load is balanced.

For case 2, the fundamental and $6k \pm 1$ injections and flows are the same as case 1. The triplen harmonics injected and the corresponding flows are SUSs and are SCS^Zs. No SCS^Zs appear in the L–L voltages nor in the current flow in the three-wire motor section of the system.

In case 3, the injections are SBSs and are therefore single SCSs. Within the sections of the system the flows are found either to be ABSs or AUSs and they resolve to two and three SCSs, respectively. The ABSs are found in the three-wire sections whereas the AUSs appear in the four-wire sections. Therefore, the corresponding SCS flows are different to that which is injected.

The case 4 results for the fundamental and $6k \pm 1$ harmonics are the same as case 3. The triplen harmonic SUSs give rise to ABSs and AUSs within the system and appear in the same sections as case 3. When compared to case 2, the results show that triplen harmonics are not purely SCS^Zs. In systems having an unbalanced harmonic source and an unbalanced load, the triplen harmonic flows can resolve to either two SCSs (SCS^+ and SCS^-) or to three SCSs (SCS^+, SCS^- and SCS^Z).

Conclusions

The table and paragraph given in Ref. 1 does not adequately describe the relationship between harmonics and symmetrical components, especially harmonic sources and does not cover unbalanced loads.

In general authors of power system literature ignore the symmetrical set concept as opposed to the asymmetrical set. The role that the symmetrical set plays in harmonic analysis and in the modelling of harmonic sources should not be ignored.

Classically, positive, negative and zero sequence components have the same frequency. The positive, negative and zero sequence symmetrical components injected by an unbalanced harmonic source have different frequencies.

The new terminology/subclasses of three-phase sets introduced distinguish balanced and unbalanced harmonic sources from each other and are found suitable for comparing injections to harmonic flows in systems in terms of symmetrical components.

The results show that what is injected in terms of symmetrical component sets by a harmonic source is not necessarily received by the system, i.e. the harmonic flows may resolve to one, two or three symmetrical component sets and this depends upon the type of three-phase set found at a given point/node in a system.

The results show that if the harmonic source and the load is balanced, the line voltages and $6k \pm 1$ harmonic currents in the motor section are symmetrical sets. Only *one SCS* per harmonic frequency is applied to the motor.

If the harmonic source is unbalanced and the load remains balanced the line voltages and currents in the motor section are unaffected by the injected triplen harmonics.

If the source is balanced and the load unbalanced, the $6k \pm 1$ harmonics in the line voltages and currents are asymmetrical balanced sets, therefore *two SCSs* per harmonic frequency (a positive and a negative sequence symmetrical component set) is applied to the motor.

If the source and load are both unbalanced, the heating effect on the motor is further increased as triplen harmonics as asymmetrical balanced sets, are added to the $6k \pm 1$ harmonic asymmetrical balanced sets resulting in the motor being subjected to *two SCSs per harmonic frequency* (a positive and a negative sequence symmetrical component set) at triplen and $6k \pm 1$ harmonic frequencies adding to the decline in the life expectancy of the motor.

The results show that triplen harmonics as flows act like $6k \pm 1$ harmonics and are not purely zero sequence, as is commonly believed, but resolve to *two SCSs per harmonic frequency* (a positive and a negative sequence symmetrical component set) in the three-wire sections and to *three SCSs per harmonic frequency* (a positive, a negative and a zero sequence symmetrical component set) in the four-wire sections when the source and load are unbalanced.

The results show that both symmetrical sets and asymmetrical sets and their resolution to symmetrical component sets are important analytical tools for harmonic analysis of systems having balanced and unbalanced harmonic sources and loads.

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