Transmission line performance with voltage sensitive loads

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Abstract In this paper, transmission line performance with voltage sensitive loads is studied. Three types of load, namely constant power, constant current and constant impedance loads are considered, individually as well as in a mixed combination, and it is shown that the transmission line loss is highest with the constant power load and lowest with the constant impedance load. The line loss for mixed load is higher than that for constant power load when the constant power component is more pronounced in the mixed load.

Keywords Long-line model; transmission line performance; voltage-sensitive load

Transmission lines constitute the arteries of an electric power system. The availability of a well-developed, high capacity system of transmission lines makes it technically and economically feasible to move large blocks of power over long distances. Usually, transmission line performance is studied by considering constant power loads. The variation of power and reactive power taken by a load with various voltages is of importance when considering the manner in which such loads are represented in transmission line studies. Most textbooks[1,2] include transmission line performance studies assuming constant power loads at the receiving end. The objective of the present paper is to incorporate the effects of the load characteristics into long-line theory to determine transmission line performance. Although classical power flow load models which are functions of voltage have been used in production grade programs for load flow studies for years,[3] not much work has been done to study the performance of a single transmission line with voltage-sensitive loads. Since a transmission line is an important component of the power system, the effects of voltage sensitive loads on its performance could form a part of the syllabus while presenting the transmission line theory in undergraduate classes. The work presented in this paper is based on the authors' previous work[4] on line losses and shunt compensation of EHV compensated transmission systems.

For constant power factor load, the active and reactive power demands, depending on the type of load, may remain constant with the voltage, change linearly with the voltage, or change as a function of the voltage squared. Constant impedance loads[1,2] such as water heaters, electric ranges, series inductors and shunt capacitors are represented by RLC circuits and the active and reactive powers consumed by such loads vary as the square of the voltage. The active power consumed by a lighting load containing incandescent lamps varies with voltage approximately as

V.[1,6] The active power consumed by a lighting load consisting of fluorescent lamps depends only slightly on voltage. Lighting load consumes no reactive power. The active power consumed by a synchronous motor is approximately constant with change in voltage. For induction motors, the PV and QV characteristics are determined from the equivalent circuit, assuming shaft load to remain constant.

Except in isolated cases, such as very large induction motor drives in refineries and steel mills, it is the composite load which is of interest. The composite load may include some elements of lighting and domestic loads, motor load, inverter and rectifier loads as well as transformer and cable losses. A typical composite load[2] can have the following approximate composition:

Induction motors, 60%
Synchronous motors, 20%
Other ingredients, 20%

For such a load, the real power demand is proportional to the voltage, and the reactive power demand is proportional to V.[1,3] Representation of a composite load as a constant current consuming device, where the active and reactive powers are assumed to be proportional to the voltage, gives a good approximation to real-life conditions.

Transmission line theory with voltage sensitive loads

The complex power at the receiving end of the transmission line is

$$S_r = P_r - jQ_r \tag{1}$$

For constant power factor angle, $\phi = \tan^{-1}(Q_o/P_o)$,

$$P_r = P_o \quad \text{and} \quad Q_r = Q_o \quad \text{for constant power load} \tag{2}$$

$$P_r = P_o V_r \quad \text{and} \quad Q_r = Q_o V_r \quad \text{for constant currrent load} \tag{3}$$

$$P_r = P_o V_r^2 \quad \text{and} \quad Q_r = Q_o V_r^2 \quad \text{for constant impedance load} \tag{4}$$

A mixed load containing fractions a, b and c of constant power, constant current, and constant impedance loads respectively is represented as follows:

$$P_r = \left(a + bV_r + cV_r^2\right)P_o \quad \text{and} \quad Q_r = \left(a + bV_r + cV_r^2\right)Q_o \tag{5}$$

The generalised line equations are

$$\mathbf{V_s} = \mathbf{AV_r} + \mathbf{BI_r} \tag{6}$$

$$\mathbf{I_s} = \mathbf{CV_r} + \mathbf{AI_r} \tag{7}$$

where

$$\mathbf{A} = A\angle\alpha,\ B = B\angle\beta \text{ and } C = C\angle\gamma.$$

The receiving end load current $\mathbf{I_r}$ lags $\mathbf{V_r}$ by ϕ where $\mathbf{V_r}$ is the reference phasor. $\mathbf{V_s}$ leads V_r by δ.

$$P_r = V_r I_r \cos\phi \tag{8}$$

and

$$Q_r = V_r I_r \sin\phi \tag{9}$$

Also,

$$\mathbf{I_r} = I_p - jI_q \tag{10}$$

$$I_r^2 = I_p^2 + I_q^2 \tag{11}$$

where

$$I_p = P_r/V_r \quad \text{and} \quad I_q = Q_r/V_r$$

For a given transmission line, **A** and **B** are known and V_s is held constant at 1 p.u. for all types of load. The receiving end voltage is taken as the reference phasor, and is determined for specified values of P_o and Q_o by using the following equation:[1]

$$V_s^2 = A^2V_r^2 + B^2\left(P_r^2 + Q_r^2\right)/V_r^2 + 2ABP_r\cos(\alpha - \beta) - 2ABQ_r\sin(\alpha - \beta) \tag{12}$$

The values of P_r and Q_r are substituted from equations (2)–(5) respectively for the particular type of voltage-sensitive load. The load power factor angle is assumed to remain constant at $\phi = \tan^{-1}(Q_o/P_o)$ for all types of load. Equation (12) assumes the forms given below for the various types of load.

Constant power load

$$a_1V_r^4 + b_1V_r^2 + c_1 = 0 \tag{13}$$

where

$$a_1 = A^2$$

$$b_1 = 2ABP_o\cos(\alpha - \beta) - 2ABQ_o\sin(\alpha - \beta) - V_s^2$$

$$c_1 = B^2\left(P_o^2 + O_o^2\right)$$

V_r is obtained from eqn (13).

The receiving end current I_r is

$$I_r = (1/V_r)\sqrt{\left(P_o^2 + Q_o^2\right)} \tag{14}$$

The load power factor angle is

$$\phi = \tan^{-1}(Q_o/P_o) \tag{15}$$

Then,

$$\mathbf{I_r} = I_r\cos\phi\text{-}j\,I_r\sin\phi \tag{16}$$

The load angle δ is given by

$$\delta = \tan^{-1}\left[\left(AV_r\sin\alpha + BI_r\sin(\beta - \phi)\right]/\left[\left(AV_r\cos\alpha + BI_r\cos(\beta - \phi)\right]\right. \tag{17}$$

The sending end current $\mathbf{I}_s$ is found from eqn (7).

$$\mathbf{I}_s = I_s \angle -\theta$$

$$I_s = \sqrt{\left[\left(C^2 V_r^2 + A^2 I_r^2 + 2ACV_r I_r \cos(\gamma + \alpha - \phi)\right)\right]} \tag{18}$$

$$\theta = -\tan^{-1}\left[CV_r \sin\gamma + AI_r \sin(\alpha - \phi)\right] / \left[CV_r \cos\gamma + AI_r \cos(\alpha - \phi)\right] \tag{19}$$

The sending end real and reactive powers are found from

$$P_s = V_s I_s \cos(\delta - \theta) \tag{20}$$

$$Q_s = V_s I_s \sin(\delta - \theta) \tag{21}$$

The transmission line loss P_{loss} is calculated from

$$P_{loss} = P_s - P_o \tag{22}$$

Constant current load

$$a_2 V_r^2 + b_2 V_r + c_2 = 0 \tag{23}$$

where

$$a_2 = A^2$$

$$b_2 = 2ABP_o \cos(\alpha - \beta) - 2ABQ_o \sin(\alpha - \beta)$$

$$c_2 = B^2\left(P_o^2 + Q_o^2\right) - V_s^2$$

V_r is obtained from eqn (23).

The receiving end current I_r is

$$I_r = \sqrt{\left(P_o^2 + Q_o^2\right)} \quad \text{and} \quad \phi = \tan^{-1}\left(Q_o / P_o\right) \tag{24}$$

Note that the magnitude of I_r has a different value than that for the constant power load. Using these new values of V_r and I_r, the rest of the quantities, namely, θ, δ, I_s, P_s, and Q_s are calculated, holding V_s constant at 1 p.u., using the same equations as given for the constant power load.

The transmission line loss is calculated from

$$P_{loss} = P_s - P_o V_r \tag{25}$$

Constant impedance load

$$a_3 V_r^2 + b_3 V_r + c_3 = 0 \tag{26}$$

where

$$a_3 = \sqrt{\left[A^2 + B^2\left(P_o^2 + Q_o^2\right) + 2ABP_o \cos(\alpha - \beta) - 2ABQ_o \sin(\alpha - \beta)\right]}$$

$$b_3 = 0$$

$$c_3 = -V_s^2$$

V_r is obtained from eqn (26).

Here, the receiving end current I_r is

$$I_r = V_r\sqrt{\left(P_o^2 + Q_o^2\right)} \quad \text{and} \quad \phi = \tan^{-1}\left(Q_o/P_o\right) \tag{27}$$

Note the difference in the above value of I_r from those for the constant power and the constant current loads respectively. Using the new values of V_r and I_r, the rest of the quantities, namely, θ, δ, I_s, P_s, and Q_s are calculated, holding V_s constant at 1 p.u., as before using the same equations as those given for the constant power load.

The transmission line loss is calculated from

$$P_{loss} = P_s - P_o V_r^2 \tag{28}$$

Mixed load

$$a_4 V_r^4 + b_4 V_r^3 + c_4 V_r^2 + d_4 V_r + e_4 = 0 \tag{29}$$

where

$$a_4 = A^2 + B^2\left(P_o^2 + Q_o^2\right)c^2 + 2ABcP_o\cos(\alpha - \beta) - 2ABcQ_o\sin(\alpha - \beta)$$

$$b_4 = 2B^2\left(P_o^2 + Q_o^2\right)bc + 2ABbP_o\cos(\alpha - \beta) - 2ABbQ_o\sin(\alpha - \beta)$$

$$c_4 = B^2\left(P_o^2 + Q_o^2\right)\left(2ac + b^2\right) + 2ABaP_o\cos(\alpha - \beta) - 2ABaQ_o\sin(\alpha - \beta) - V_s^2$$

$$d_4 = B^2\left(P_o^2 + Q_o^2\right)(2ab)$$

$$e_4 = 2B^2\left(P_o^2 + Q_o^2\right)a^2$$

The above polynomial equation is solved for V_r, using the Newton-Raphson method. Here, the receiving end current I_r, is

$$I_r = (1/V_r)\sqrt{\left[P_o^2\left(a + bV_r + cV_r^2\right)^2 + Q_o^2\left(a + bV_r + cV_r^2\right)^2\right]} \tag{30}$$

Using this value of V_r and I_r, the rest of the quantities, namely, θ, δ, I_s, P_s, and Q_s are calculated, holding V_s constant at 1 p.u., as before. The transmission line loss is calculated from

$$P_{loss} = P_s - P_o\left(a + bV_r + cV_r^2\right) \tag{31}$$

Numerical results

The performance of a 220 kV, 300 km transmission line is studied. The transmission line data[2] are given in Appendix 1. The line is operated at a sending-end voltage of 220 kV. The load power for the constant power load is taken as P_o = 135 MW, Q_o = –5.7 MVAR. The values of P_o and Q_o are assumed to have the same values for the voltage sensitive loads as those for the constant power load.

Table 1 gives the line performance results. It is shown that

- for a constant power load, the quantities, namely, voltage regulation, load current, load angle, and sending-end current, have the highest values, and for the constant impedance load, they have the lowest values among the three types of load.

TABLE 1 Comparison of transmission line performance for voltage sensitive loads for V_s = 220 kV, P_o = 135 MW, Q_o = −5.7 MVAR and constant power factor angle = −2.417 degree

Line Performance Parameters	Constant power load	Constant current load	Constant impedance load	Mixed load
Load Voltage V_r (kV)	192.798	201.685	205.474	199.748
Load Current I_r (kA)	0.405	0.355	0.331	0.391
Load angle δ (deg)	28.853	25.146	21.375	27.331
Sending-end current I_s (kA)	0.409	0.366	0.334	0.373
θ (deg)	13.140	16.151	17.798	14.291
Sending-end power P_s (MW)	149.985	137.721	127.157	138.467
Sending-end reactive power Q_s (MVAR)	42.212	21.808	7.952	32.083
Voltage-dependent load power P_r (MW)	135.000	123.761	117.761	126.530
Voltage-dependent reactive power Q_r (MVAR)	−5.700	−5.225	−4.972	−5.342
Power loss P_{loss}(MW)	14.985	13.960	9.396	11.937

- the power loss is the highest for the constant power load and the least for the constant impedance load among the three types of load.
- the power loss for mixed load is higher than that for the constant power load when the mixed load contains a pronounced component of constant power load.

Conclusion

The effects of the load characteristics on the performance of a 220 kV transmission line are studied in this paper. For a fixed sending end voltage and a fixed load power, it is shown that the voltage regulation, the load angle and the transmission line loss have widely differing values which are dependent on the type of voltage-sensitive load connected to the line.

References

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Appendix

220 kV line data
Resistance of line = 0.093 Ω km^{-1}
Inductance of line = 1.33 m Henry km^{-1}
Capacitance of line = 8.86 nanofarads km^{-1}
Frequency = 60 hertz