Fundamental loop-current method using 'virtual voltage sources' technique for special cases

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Abstract A new technique based on the use of virtual voltage sources makes any electric circuit solvable by the fundamental loop-current method in an easy and formulated way for students. Thus, the fundamental loop-current method is systematised (especially for nonplanar circuits) and the difficulties presented up to now for special circuit categories cease to exist.

Keywords fundamental loop; link; non-convertible current source; special cases; tree; virtual voltage source

Solving an electric circuit in a systematic way demands topological concepts like those referred to in many textbooks on electric circuits.[1,2,3] The most important concepts are the graph, the tree and the fundamental loop.

To each electric circuit corresponds a graph, which is the geometric figure resulting from the circuit, when each branch of a non-active circuit element is substituted by a line part or a curved part; the circuit voltage sources are short-circuited and the current sources are open-circuited.

If a graph can be drawn on a plane in such a way that any two of its branches are not to be crossed at any point that is not a circuit node, then this graph is named a planar graph and the circuit, a planar circuit (Fig. 1(a, b)). Otherwise, they are called a nonplanar graph and a nonplanar circuit, respectively (Fig. 1(c)).

A graph for which the transition from a node to another node is made by following continuous branches is called a connected graph (Figure 2(a)). In the opposite case, it is called a non-connected graph (Figure 2(b)).

Given a connected graph for a circuit, a tree (T) is a connected part of the graph which contains all the nodes and simultaneously does not contain any loops. The branches of a tree are named tree branches and the rest of the branches (constituting the graph) are called links (LT) or chords.

A basic property resulting directly from the definition of the tree is that if a graph has n nodes, then the tree has n nodes as well, but it has $n - 1$ branches.

Figure 3 shows two trees T_1 and T_2 of a connected graph and their links LT_1 and LT_2, respectively.

Obviously, the possible trees of a connected graph result from all the combinations of the graph branches that satisfy the tree properties.

For a given tree in a connected graph, a fundamental loop (FL) is each loop which

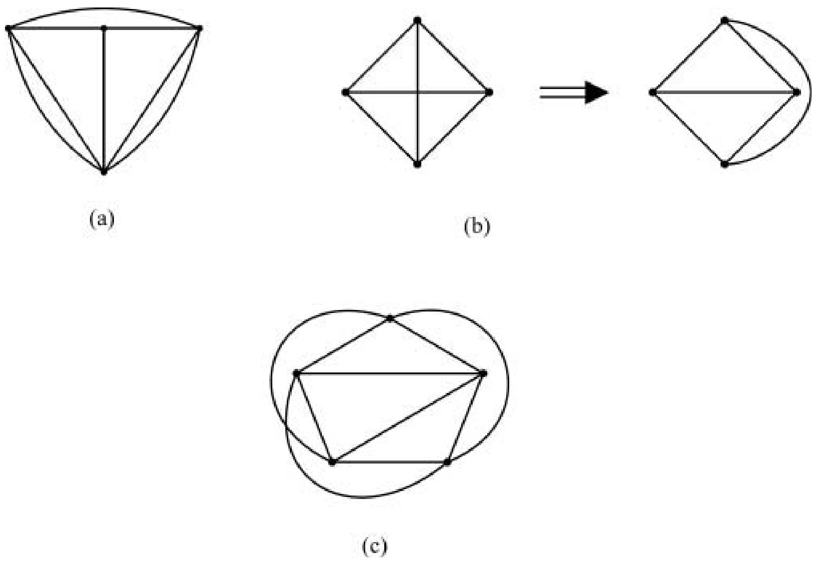

Fig. 1 *(a), (b) Planar graphs; (c) non-planar graph.*

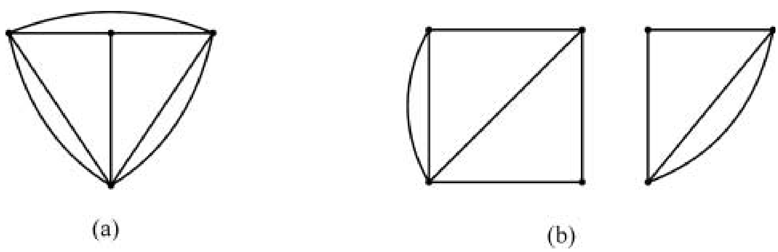

Fig. 2 *(a) Connected graph; (b) non-connected graphs.*

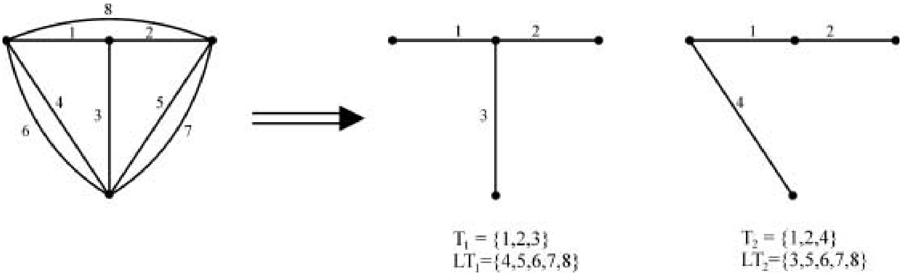

Fig. 3 *Trees of a connected graph.*

includes only one link, while all the other branches of this tree are tree branches of it.

Figure 4 shows the fundamental loops of a tree selected from a connected graph.

The analysis of d.c. circuits by the fundamental loop-current method is based on the above-mentioned topological concepts and applies in general to connected planar or nonplanar circuits, in which most of their sources are voltage sources.[1,2,5]

This method is based on two theorems:

- The number m of the links (and therefore of the fundamental loops) of a tree of a connected graph having b branches and n nodes is $m = b - n + 1$.
- The equations resulting from applying Kirchhoff's voltage law to each fundamental loop of a tree of a connected graph are independent of each other.

Also, this method is relatively easy and systematic for circuits that contain only independent voltage sources.

The difficulty of this method – from a systematic standardisation and a pedagogical effectiveness point of view – starts when the circuit also contains dependent sources. It is most difficult when there are current sources (independent, dependent) which are not transformable into voltage sources (independent and dependent, respectively), because there are no resistances (at least in an obvious way) parallel to them.

The problem of the non-convertible current sources is usually tackled by the use of supermeshes and only for planar circuits. For nonplanar circuits, the problem is not confronted at all by using loops; the only method that can be used is the node-voltage method (where supernodes are used when there are non-convertible voltage

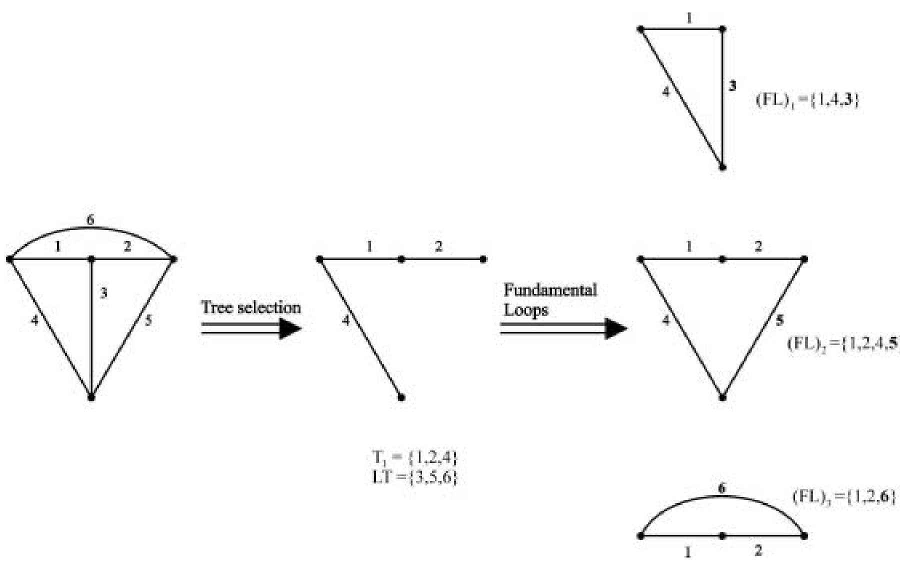

Fig. 4 *Fundamental loops for a selected tree of a connected graph.*

sources).[2,4,5] However, the use of supermeshes or supernodes is something that students are not able easily to understand and apply; more specifically, the generalisation and standardisation involved in dealing with special cases in electric circuits is not easy for them.

This paper, beyond the effort of facing the fundamental loop-current method in a systematic way (especially for nonplanar circuits) also faces the problem of the non-convertible current sources by introducing the concept of virtual voltage sources.

The term virtual voltage source means that a non-convertible current source (independent or dependent) is substituted by a voltage source (independent or dependent respectively), which has a value equal to the voltage at its terminals and which is obviously unknown.

Fundamental loop-current method

Facing connected planar and nonplanar circuits in a systematic and standard way using the fundamental loop-current method depends on the kind of sources that exist in the circuit and also on whether the existing current sources are convertible or non-convertible. Based on all these, the fundamental loop-current method can be examined for four different cases.

Case A. Connected planar or nonplanar circuit with independent (voltage or current) sources but with all possibly existing current sources convertible

In such a case, the current sources are initially transformed into voltage sources and then in the resulting equivalent circuit the following steps are executed:

Step 1. From the graph of the circuit, a tree is selected which, as is known, has m links. Arranging each time a link, a fundamental loop results. Thus, m fundamental loops are found that correspond to the specific tree which is selected.

Step 2. In all the m fundamental loops, the fundamental loop currents i_1, i_2, i_3, . . . , i_m are defined, having any direction.

Step 3. The fundamental loop equations are written in matrix form as follows:

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & . & . & . & R_{1m} \\ R_{21} & R_{22} & R_{23} & . & . & . & R_{2m} \\ R_{31} & R_{32} & R_{33} & . & . & . & R_{3m} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ R_{m1} & R_{m2} & R_{m3} & . & . & . & R_{mm} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ . \\ . \\ . \\ i_m \end{bmatrix} = \begin{bmatrix} \Sigma v_1 \\ \Sigma v_2 \\ \Sigma v_3 \\ . \\ . \\ . \\ \Sigma v_m \end{bmatrix}$$

where: R_{ii}, $\forall i = 1, 2, 3, \ldots, m$ denotes the self-resistance of the $(FL)_i$ and is equal to the sum of all resistances of this fundamental loop.

$R_{ij} = R_{ji}$, $\forall i \neq j$, $i, j = 1, 2, 3, \ldots, m$ denotes the mutual resistance of the $(FL)_i$ and $(FL)_j$ and is equal to the sum of all resistances in the common branches of these fundamental loops. Its sign is (+) if the loop current directions on the common branches are the same, otherwise is (–).

Σv_i, $\forall i = 1, 2, 3, \ldots, m$ is the algebraic sum of the values of all voltage sources of the $(FL)_i$. The values of those sources whose loop current goes from the negative to the positive pole are taken to be positive; otherwise, they are taken to be negative.

Step 4. The resulting $m \times m$ linear system is solved using the Cramer method or by the matrix inversion method and the currents $i_1, i_2, i_3, \ldots, i_m$ are thus known.

Step 5. The currents of all branches are calculated combining the fundamental loop currents and as a consequence the voltages of all circuit elements are known.
In other words, the solution of the electric circuit is completed.

Notes

- The resistance matrix is symmetrical since $R_{ij} = R_{ji}$, $\forall i \neq j$.
- There is no particular reason to take the fundamental loop currents in the same direction, since the sign of the R_{ij}, $\forall i \neq j$ is not known from the beginning as in the mesh current method, but attention must be paid to it.

Example
For the circuit of Fig. 5, using the fundamental loop method show that the power developed is equal to the power dissipated.

Solution. Selecting a tree from the nonplanar graph of the circuit, the fundamental loops are found corresponding to it and their currents are defined (Fig. 6).

The fundamental loop equations in matrix form are:

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} \Sigma v_1 \\ \Sigma v_2 \\ \Sigma v_3 \\ \Sigma v_4 \\ \Sigma v_5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 15 & 12 & 9 & -9 & -5 \\ 12 & 18 & 7 & -9 & -3 \\ 9 & 7 & 16 & -4 & -5 \\ -9 & -9 & -4 & 17 & 0 \\ -5 & -3 & -5 & 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \\ 0 \\ -16 \\ -15 \end{bmatrix} \Rightarrow \begin{matrix} i_1 = -8.31\,\text{A} \\ i_2 = 2.73472\,\text{A} \\ i_3 = -0.82217\,\text{A} \\ i_4 = -4.08625\,\text{A} \\ i_5 = -10.49134\,\text{A} \end{matrix}$$

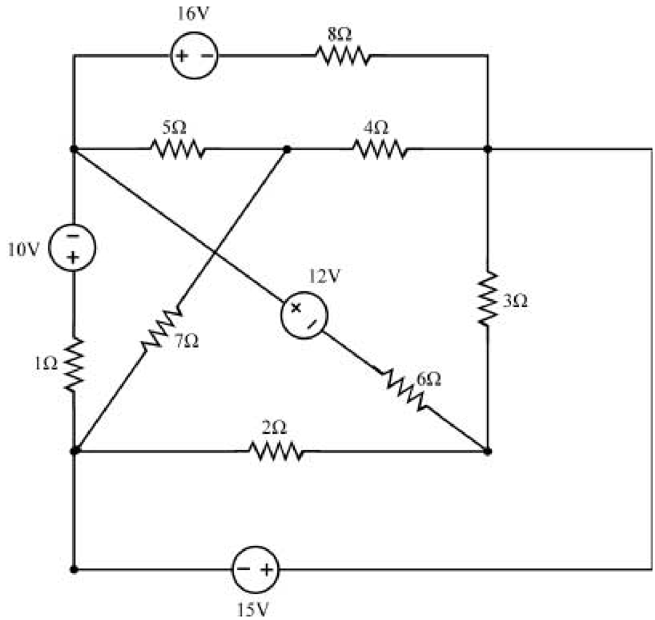

Fig. 5 *Circuit for case A.*

Hence,

$$p_{(16\text{V})} = 16 \cdot i_4 = -65.38\,\text{W} \quad \text{(delivered)}$$

$$p_{(10\text{V})} = 10 \cdot i_1 = -83.1\,\text{W} \quad \text{(delivered)}$$

$$p_{(15\text{V})} = 15 \cdot i_5 = -157.3701\,\text{W} \quad \text{(delivered)}$$

$$p_{(12\text{V})} = 12 \cdot i_2 = -32.81664\,\text{W} \quad \text{(delivered)}$$

$$p_{\text{RESIST.}} = 5(i_1 + i_2 - i_4)^4 + 4(i_1 + i_2 + i_3 - i_4)^2 + 3(i_1 + i_2 + i_3 - i_5)^2 + 2(i_1 + i_3 - i_5)^2 + 1i_1^2 + 6i_2^2 + 7i_2^2 + 8i_4^2 = 338.666\ \text{W}$$

Thus,

$$\left.\begin{array}{r} p_{\text{DELIV.}} = p_{(16\text{V})} + p_{(10\text{V})} + p_{(15\text{V})} + p_{(12\text{V})} = 338.666\,\text{W} \\ p_{\text{DISSIP.}} = p_{\text{RESIST.}} = 338.666\,\text{W} \end{array}\right\} \Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

Case B. Connected planar or nonplanar circuit with independent (voltage or current) sources but, with at least one current source not transformable to voltage source or for which the transformation is difficult (special case)

In such a case, the following steps are executed:

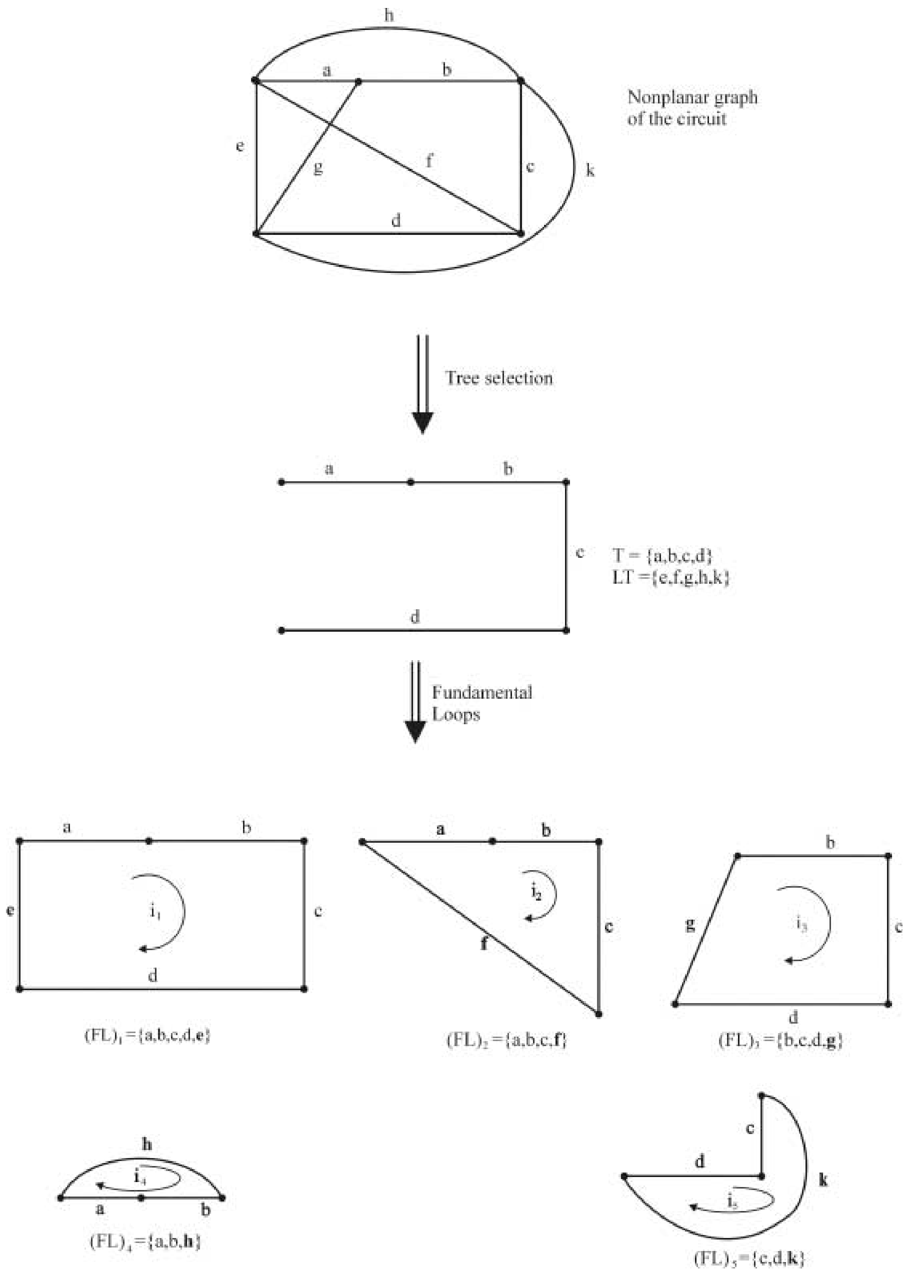

Fig. 6 *Currents of fundamental loops for circuit shown in Fig. 5.*

Step 1. In the locations of the current sources that are non-convertible, virtual voltage sources are considered with values equal to the corresponding voltage values present at the terminals of the non-convertible current sources of the given circuit.

Step 2. From the virtual graph of the circuit, a tree is selected which has, as it is well known *m* links. Placing each time one link, a fundamental loop results. Thus, *m* fundamental loops are found corresponding to the specific selected tree.

Step 3. In all the *m* fundamental loops, the fundamental loop currents i_1, i_2, i_3, . . . , i_m are defined having any direction.

Step 4. The fundamental loop equations are written in matrix form as in case A. However, in this case the virtual voltage sources are similarly taken as the existing (non-virtual) ones and they are included in the terms Σv_i.

Step 5. For each virtual voltage source, an equation is introduced in the matrix that describes the corresponding non-convertible current source with a linear combination of the unknown fundamental loop currents of the problem, eliminating each time an equation that contains a virtual voltage. The remaining equations needed for the solution are taken from the initial form of the matrix, as they are (those that do not contain other unknown currents than the fundamental loop currents) or as they result after the appropriate additions or subtractions in order to eliminate the virtual voltages appearing initially.

By doing so, the new matrix form of the equations does not any longer represent Ohm's Law, but it is simply an algebraically $m \times m$ equivalent system, which can lead to the finding of the fundamental loop currents.

Step 6. The resulting $m \times m$ linear system is solved as in the previous case and so the fundamental loop currents are readily available.

Step 7. The currents of all branches are calculated by combining the fundamental loop currents, and as a consequence, the voltages of all circuit elements are known, except those at the terminals of the non-convertible current sources (that is the virtual voltages). The calculation of these voltages is done using the equations eliminated from the initial form of the system, since the fundamental loop currents are already known.

In other words, the solution of the electric circuit is completed.

Example

For the circuit of Fig. 7, using the fundamental loop-current method show that the power developed is equal to the power dissipated.

Solution. Since the independent current sources of the 2 A and 3 A are non-convertible (there are no resistances parallel to them), it is considered that, instead

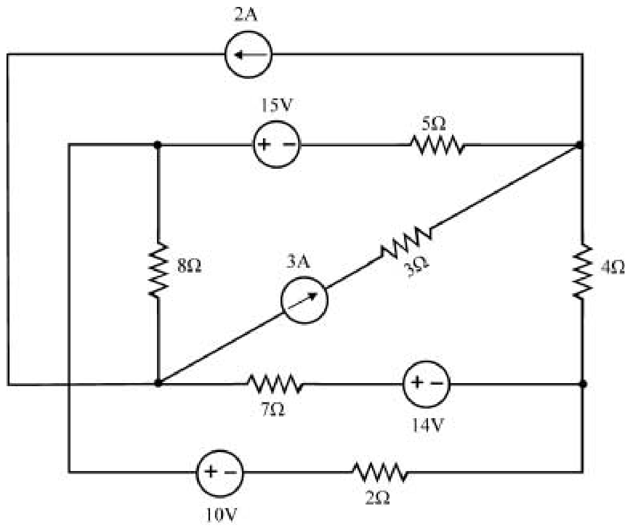

Fig. 7 *Circuit for case B.*

of them, virtual voltage sources exist at their location with values v_x and v_y equal to the voltages at their terminals respectively.

By selecting a tree from the virtual nonplanar graph of the circuit, the fundamental loops are found corresponding to this tree and their currents are defined (Fig. 8).

The fundamental loop equations in matrix form are:

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \Sigma v_1 \\ \Sigma v_2 \\ \Sigma v_3 \\ \Sigma v_4 \end{bmatrix} \Rightarrow \begin{bmatrix} 24 & 11 & 11 & 9 \\ 11 & 14 & 11 & 4 \\ 11 & 11 & 11 & 4 \\ 9 & 4 & 4 & 11 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -1 \\ v_y + 14 \\ 14 - v_x \\ -5 \end{bmatrix} \quad (1)$$

Substituting the first line of relationship (1) by the equation $i_3 = -2\,\text{A}$ which applies to the current source of 2 A, the second line by the equation $i_2 = 3\,\text{A}$ which applies to the current source of 3 A, the third line by the first and the fourth line as is, the following algebraically equivalent system results:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 24 & 11 & 11 & 9 \\ 9 & 4 & 4 & 11 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \\ -5 \end{bmatrix} \Rightarrow \begin{matrix} i_1 = -0.27869\,\text{A} \\ i_2 = 3\,\text{A} \\ i_3 = -2\,\text{A} \\ i_4 = -0.59016\,\text{A} \end{matrix}$$

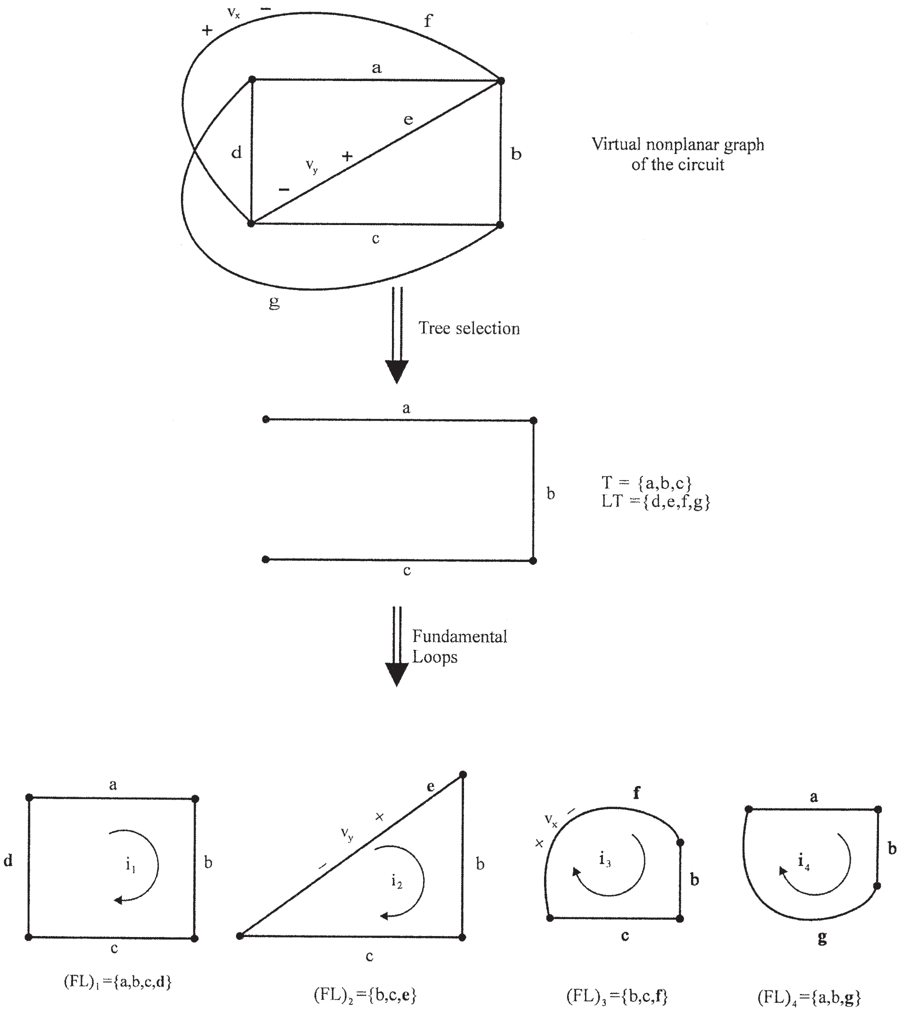

Fig. 8 *Currents of fundamental loops for circuit shown in Fig. 7.*

In order to find the voltages at the non-convertible current source terminals, the following are considered:

From the third line of relationship (1) results:

$$14 - v_x = 11i_1 + 11i_2 + 11i_3 + 4i_4 \Rightarrow v_x = 8.42623\,\text{V}$$

From the second line of relationship (1) results:

$$v_y + 14 = 11i_1 + 14i_2 + 11i_3 + 4i_4 \Rightarrow v_y = 0.57377\,\text{V}$$

Hence,

$$p_{(3\text{A})} = -3 \cdot v_y = -1.72131\,\text{W} \quad \text{(delivered)}$$

$$p_{(2\text{A})} = -2 \cdot v_x = -16.85246\,\text{W} \quad \text{(delivered)}$$

$$p_{(15\text{V})} = 15 \cdot (i_1 + i_4) = -13.03275\,\text{W} \quad \text{(delivered)}$$

$$p_{(10\text{V})} = -10 \cdot i_4 = 5.9016\,\text{W} \quad \text{(dissipated)}$$

$$p_{(14\text{V})} = -14 \cdot (i_1 + i_2 + i_3) = -10.09834\,\text{W} \quad \text{(delivered)}$$

$$p_{\text{RESIST}} = 5(i_1 + i_4)^2 + 4(i_1 + i_2 + i_3 + i_4)^2 + 7(i_1 + i_2 + i_3)^2 + 8i_1^2 + 3i_2^2 + 2i_4^2 = 35.80324\,\text{W}$$

Thus,

$$\left.\begin{aligned} p_{\text{DELIV.}} &= p_{(3\text{A})} + p_{(2\text{A})} + p_{(15\text{V})} + p_{(14\text{V})} = 41.70486\,\text{W} \\ p_{\text{DISSIP.}} &= p_{\text{RESIST.}} + p_{(10\text{V})} = 41.70486\,\text{W} \end{aligned}\right\} \Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

Case C. Connected planar or nonplanar circuit with independent and dependent (voltage or current) sources, but with all current sources (independent and dependent) that possibly exist in the circuit convertible

In this case, the current sources (independent and dependent) are initially transformed into voltage sources (independent and dependent respectively), and then in the resulting equivalent circuit the following steps are executed:

Step 1. From the graph of the circuit, a tree is selected which has, as is well known, *m* links. Placing each time one link, a fundamental loop results. Thus, *m* fundamental loops are found corresponding to the selected tree.

Step 2. In all the *m* fundamental loops, the fundamental loop currents i_1, i_2, i_3, . . . , i_m are defined having any direction.

Step 3. The fundamental loop equations are written in matrix form as in case A.

Step 4. The dependent quantities appearing in the matrix form are expressed with respect to the unknown fundamental loop currents. However, this implies that the unknown fundamental loop currents appear in the second part of the matrix form of the equations as well.

Step 5. The elements of the equation lines are rearranged (when needed) so that the unknown fundamental loop currents appear only in the left-hand part of the equations.

Step 6. The resulting $m \times m$ linear system is solved as in the previous cases and so the fundamental loop currents are readily available.

Step 7. The currents of all branches are calculated by combining the fundamental loop currents and as a consequence the voltages of all circuit elements are known.

In other words, the solution of the electric circuit is completed.

Notes

- A dependent current source is considered convertible when there is a resistance parallel to it and simultaneously the dependent quantity of this or other dependent source is not located at this parallel resistance. If something like this were to happen, the source transformation would result in the elimination of the dependent quantity and therefore further steps for the problem solution would be difficult or even impossible.
- An independent current source is considered convertible when there is a resistance parallel to it and simultaneously the dependent quantity of a dependent current or voltage source does not appear at this resistance or at the source (for the same reason as previously referred).

Example

For the circuit of Fig. 9, using the fundamental loop-current method show that the power developed is equal to the power dissipated.

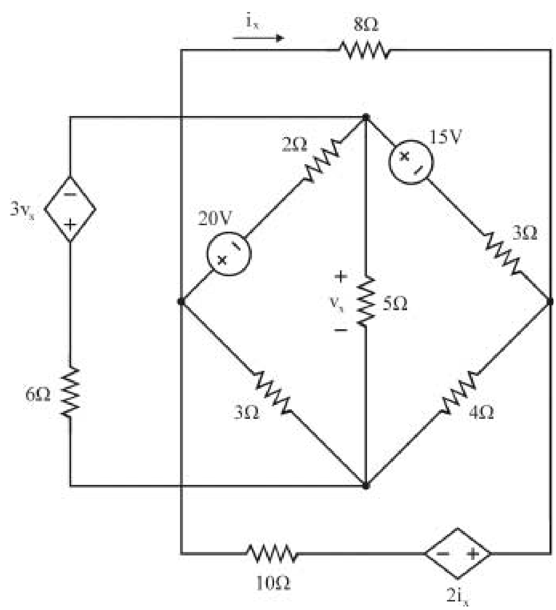

Fig. 9 *Circuit for case C.*

Solution. By selecting a tree from the nonplanar graph of the circuit, the fundamental loops corresponding to it and their currents are defined (Fig. 10). The fundamental loop equations in matrix form are:

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} \Sigma v_1 \\ \Sigma v_2 \\ \Sigma v_3 \\ \Sigma v_4 \\ \Sigma v_5 \end{bmatrix} \Rightarrow \begin{bmatrix} 12 & -5 & 9 & -9 & -5 \\ -5 & 10 & -7 & 7 & 5 \\ 9 & -7 & 19 & -11 & -5 \\ -9 & 7 & -11 & 21 & 5 \\ -5 & 5 & -5 & 5 & 11 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix}$$

$$= \begin{bmatrix} -15 \\ -20 \\ 20 \\ -20-2i_x \\ -3v_x \end{bmatrix} = \begin{bmatrix} -15 \\ -20 \\ 20 \\ -20-2i_3 \\ -15(-i_1+i_2-i_3+i_4+i_5) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 12 & -5 & 9 & -9 & -5 \\ -5 & 10 & -7 & 7 & 5 \\ 9 & -7 & 19 & -11 & -5 \\ -9 & 7 & -9 & 21 & 5 \\ -20 & 20 & -20 & 20 & 26 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -15 \\ -20 \\ 20 \\ -20 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} i_1 = -4.06855\,\text{A} \\ i_2 = -2.49146\,\text{A} \\ i_3 = 1.48429\,\text{A} \\ i_4 = -1.48429\,\text{A} \\ i_5 = 1.07037\,\text{A} \end{matrix}$$

Hence,

$$p_{(20\text{V})} = 20\cdot(i_2 - i_3 + i_4) = -109.2008\,\text{W} \quad \text{(delivered)}$$

$$p_{(15\text{V})} = 15\cdot i_1 = -61.02825\,\text{W} \quad \text{(delivered)}$$

$$p_{(2i_x)} = 2i_x \cdot i_4 = 2i_3 \cdot i_4 = -4.40623\,\text{W} \quad \text{(delivered)}$$

$$p_{(3v_x)} = 3v_x \cdot i_5 = 15(-i_1 + i_2 - i_3 + i_4 + i_5)\cdot i_5 = -5.15576\,\text{W} \quad \text{(delivered)}$$

$$p_{\text{RESIST.}} = 2(i_2 - i_3 + i_4)^2 + 4(i_1 + i_3 - i_4)^2 + 5(-i_1 + i_2 - i_3 + i_4 + i_5)^2 + 8i_3^2 + 6i_5^2 + 3i_1^2 + 3i_2^2 + 10i_4^2 = 179.79104\,\text{W}$$

Thus,

$$\left.\begin{matrix} p_{\text{DELIV.}} = p_{(20\text{V})} + p_{(15\text{V})} + p_{(2i_x)} + p_{(3v_x)} = 179.79104\,\text{W} \\ p_{\text{DISSIP.}} = p_{\text{RESIST.}} = 179.79104\,\text{W} \end{matrix}\right\} \Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

Case D. Connected planar or nonplanar circuit with independent and dependent (voltage or current) sources but, with at least one current source (independent or dependent) not transformable to voltage source (independent or dependent respectively) or for which the transformation is difficult (special case)

In such a case, the following steps are executed:

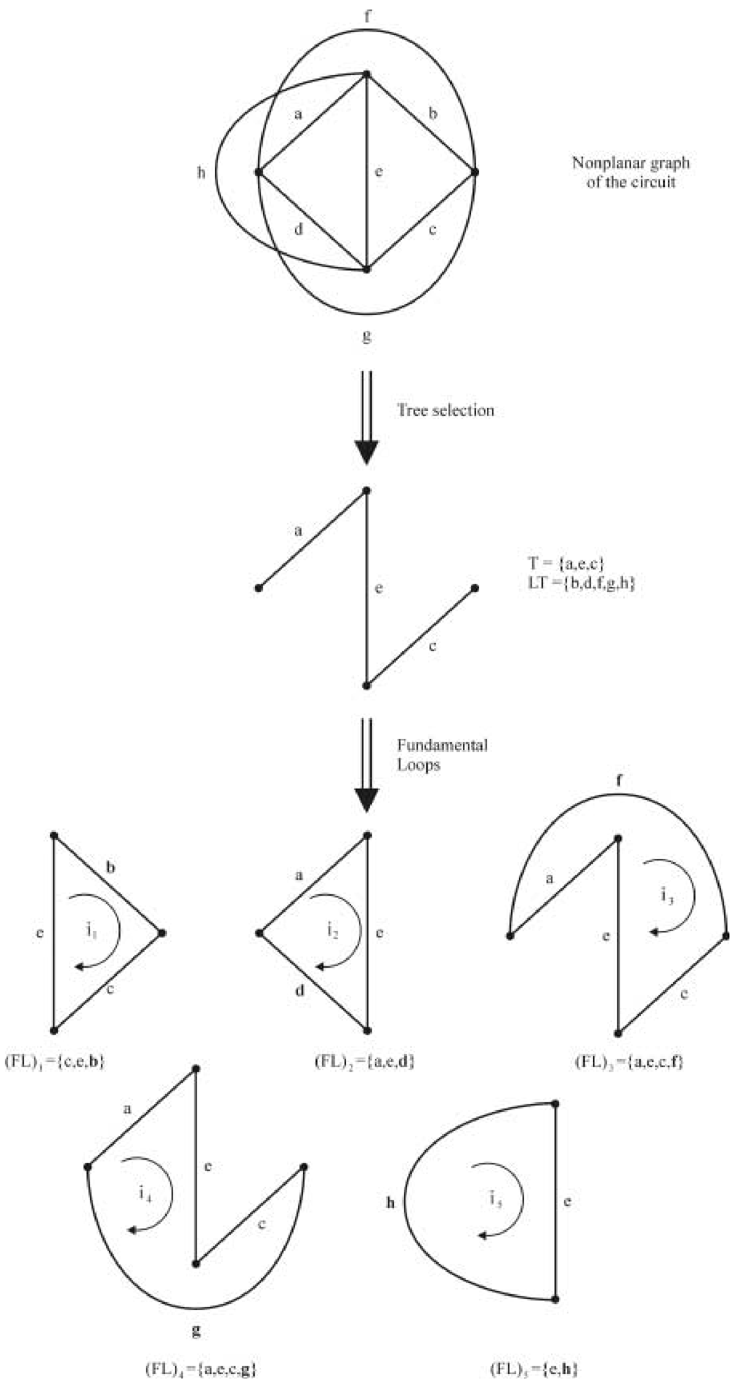

Fig. 10 *Currents of fundamental loops for circuit shown in Fig. 9.*

Step 1. In the locations of the current sources that are non-convertible, virtual voltage sources are considered with values equal to the corresponding voltage values present at the terminals of the non-convertible current sources of the given circuit.

Step 2. From the virtual graph of the circuit, a tree is selected which has, as is well know m links. Placing each time one link, a fundamental loop results. Thus, m fundamental loops are found corresponding to the selected tree.

Step 3. In all the m fundamental loops, the fundamental loop currents i_1, i_2, i_3, . . . , i_m are defined having any direction.

Step 4. The fundamental loop equations are written in matrix form as in case A. However, in this case the virtual voltage sources are similarly taken as the existing (non-virtual) ones and they are included in the terms Σv_i.

Step 5. For each virtual voltage source, an equation is introduced in the matrix that describes the corresponding non-convertible current source with a linear combination of the unknown fundamental loop currents, eliminating each time an equation that contains a virtual voltage. The remaining equations needed for the solution are taken from the initial form of the matrix, as they are (those that do not contain other unknown currents than the fundamental loop currents) or as they result after the appropriate additions or subtractions in order to eliminate the virtual voltages appearing initially. By doing so, the new matrix form of the equations does not any longer represent Ohm's Law, but it is simply an algebraically $m \times m$ equivalent system, which can lead to the finding of the fundamental loop currents.

Step 6. The dependent quantities appearing in the matrix form are expressed with respect to the unknown fundamental loop currents. However, this implies that the unknown fundamental loop currents appear in the second part of the matrix form of the equations as well.

Step 7. The elements of the equation lines are rearranged so as the unknown fundamental loop currents appear only in the left-hand part of the equations.

Step 8. The resulting $m \times m$ linear system is solved as in case A and so the fundamental loop currents are readily available.

Step 9. The currents of all branches are calculated combining the fundamental loop currents and as a consequence the voltages of all circuit elements are known, except those at the terminals of the non-convertible current sources (that is the virtual voltages). The calculation of these voltages is done using the equations that were eliminated from the initial form of the system, since the fundamental loop currents are already known.

In other words, the solution of the electric circuit is completed.

Example

For the circuit of Fig. 11, using the fundamental loop-current method show that the power developed is equal to the power dissipated.

Solution. Since the independent current source of 5 A is non-convertible (there is no resistance parallel to it), it is considered that, instead of it, a virtual voltage source exists at its location with a value v_x equal to the voltage at its terminals.

Also, for the same reason, the dependent current source of $2v_\varphi$ is non-convertible and therefore a virtual voltage source is considered to exist at its location with a value v_y equal to the voltage at its terminals.

By selecting a tree from the nonplanar graph of the circuit, the fundamental loops are found corresponding to this tree and their currents are defined (Fig. 12).

The fundamental loop equations in matrix form are:

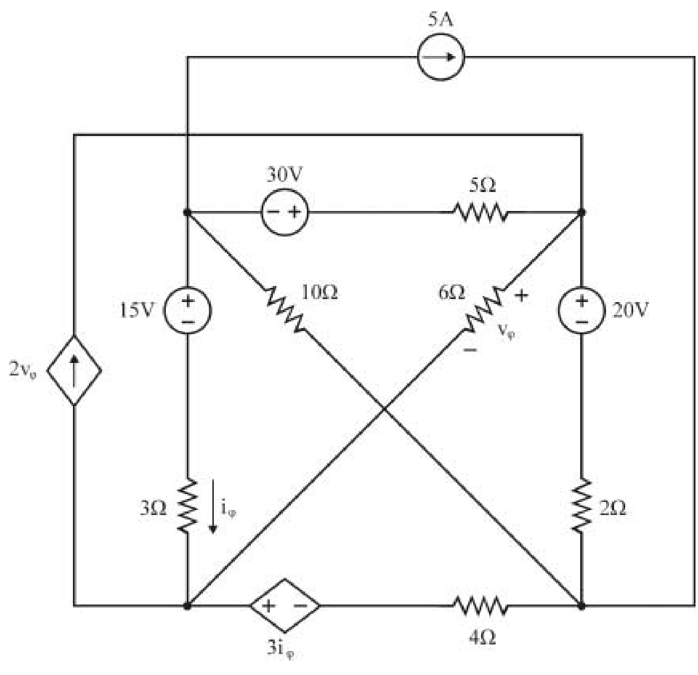

Fig. 11 *Circuit for case D.*

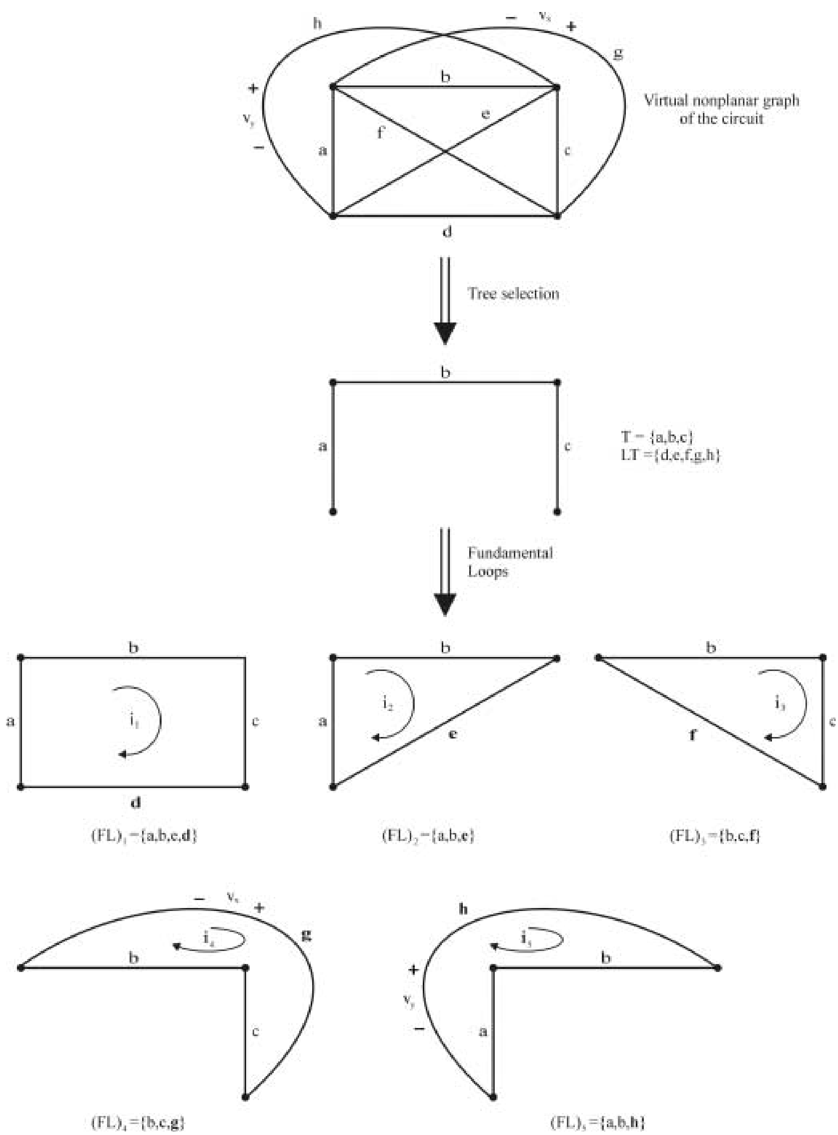

Fig. 12 *Currents of fundamental loops for circuit shown in Fig. 11.*

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} \Sigma v_1 \\ \Sigma v_2 \\ \Sigma v_3 \\ \Sigma v_4 \\ \Sigma v_5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 14 & 8 & 7 & -7 & -8 \\ 8 & 14 & 5 & -5 & -8 \\ 7 & 5 & 17 & -7 & -5 \\ -7 & -5 & -7 & 7 & 5 \\ -8 & -8 & -5 & 5 & 8 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 25+3i_\varphi \\ 45 \\ 10 \\ -10+v_x \\ -45+v_y \end{bmatrix} \tag{2}$$

Substituting the first line of relationship (2) by the equation $i_4 = 5$ A which applies to the independent current source of 5 A, the second line by the equation $i_5 = 2v_\varphi$ which applies to the dependent current source of $2v_\varphi$, the third line by the first, the fourth line by the second and the fifth line by the third, the following algebraically equivalent system results:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 14 & 8 & 7 & -7 & -8 \\ 8 & 14 & 5 & -5 & -8 \\ 7 & 5 & 17 & -7 & -5 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 2v_\varphi \\ 25+3i_\varphi \\ 45 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 12i_2 \\ 25+3(-i_1-i_2+i_5) \\ 45 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & -12 & 0 & 0 & 1 \\ 17 & 11 & 7 & -7 & -11 \\ 8 & 14 & 5 & -5 & -8 \\ 7 & 5 & 17 & -7 & -5 \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 25 \\ 45 \\ 10 \end{bmatrix} \Rightarrow \begin{matrix} i_1 = -8.27166\text{A} \\ i_2 = -1.60897\text{A} \\ i_3 = 0.84754\text{A} \\ i_4 = 5\text{A} \\ i_5 = -19.30764\text{A} \end{matrix}$$

In order to find the voltages at the non-convertible current source terminals the following are considered:

The fourth line of relationship (2) gives:

$$-10+v_x = -7i_1 - 5i_2 - 7i_3 + 7i_4 + 5i_5 \Rightarrow v_x = 8.47549\text{ V}$$

The fifth line of relationship (2) gives:

$$-45+v_y = -8i_1 - 8i_2 - 5i_3 + 5i_4 + 8i_5 \Rightarrow v_y = -9.65378\text{ V}$$

Hence,

$$p_{(5\text{A})} = -5 \cdot v_x = -42.37745\text{ W} \quad \text{(delivered)}$$

$$p_{(2v_\varphi)} = -2v_\varphi \cdot v_y = -12i_2 \cdot v_y = -186.39171\text{ W} \quad \text{(delivered)}$$

$$p_{(30\text{V})} = -30 \cdot (i_1 + i_2 + i_3 - i_4 - i_5) = -158.2365\text{ W} \quad \text{(delivered)}$$

$$p_{(20\text{V})} = 20 \cdot (i_1 + i_3 - i_4) = -248.4824\,\text{W} \quad \text{(delivered)}$$

$$p_{(15\text{V})} = 15 \cdot i_\varphi = 15 \cdot (-i_1 - i_2 + i_5) = -141.40515\,\text{W} \quad \text{(delivered)}$$

$$p_{(3i_\varphi)} = -3i_\varphi \cdot i_1 = -3(-i_1 - i_2 + i_5) \cdot i_1 = -233.93106\,\text{W} \quad \text{(delivered)}$$

$$p_{\text{RESIST.}} = 5(i_1 + i_2 + i_3 - i_4 - i_5)^2 + 2(i_1 + i_3 - i_4)^2 + 4i_1^2 + 3(-i_1 - i_2 + i_5)^2 + 10i_3^2 + 6i_2^2 = 1010.82484\,\text{W}$$

Thus,

$$\left.\begin{aligned} p_{\text{DELIV.}} = p_{(5\text{A})} + p_{(2v_\varphi)} + p_{(30\text{V})} + p_{(20\text{V})} + p_{(15V)} + p_{(3i_\varphi)} &= 1010.82484\,\text{W} \\ p_{\text{DISSIP.}} = p_{\text{RESIST.}} &= 1010.82484\,\text{W} \end{aligned}\right\}$$

$$\Rightarrow p_{\text{DELIV.}} = p_{\text{DISSIP.}}$$

Conclusions

The classification of connected planar or nonplanar electric circuits into four categories, as have been examined in this paper, enables the student to solve easily, following similar procedures, any circuit by the fundamental loop-current method. However, this method basically applies to nonplanar circuits for which the mesh-current method cannot be used. For planar circuits, the mesh-current method can be applied and its use is recommended, since it is an easier method than the fundamental loop-current method. Therefore, all the examples presented in this article deal with nonplanar circuits.

With respect to the special cases, the second and fourth (B, D) dealt with virtual voltage sources; this results in the non-differentiation of these cases regarding the overall methodological steps to be followed.

Another equally important advantage of using virtual voltage sources is the immediate finding of the voltages present at the terminals of the non-convertible current sources, given that the fundamental loop currents are known, since their voltages are already expressed by the way the equations are written in matrix form. So, the power developed by these sources is easy to calculate and therefore the proof of the power balance does not present any difficulties.

Special attention must be paid to the conditions under which a current source is transformed to a voltage source; this is because whereas a current source is convertible when there is a resistance parallel to it, for methodological purposes it should be considered non-convertible, when the conditions mentioned in the fourth case (D) are not met.

Finally, the fundamental loop-current method, as has been analysed already, can obviously be used for circuits in the sinusoidal steady state (a.c. circuits). However, the necessary condition is that all circuit sources are of the same frequency (otherwise the principle of superposition is used). So, if all sources are of the same frequency, the fundamental loop-current method starts after the circuit transformation to the frequency domain.

References

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