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# Teaching the self-excited induction generator using Matlab

Yaser N. Anagreh and Imadden M. Al-Refae'e

Department of Electrical Power Engineering, Yarmouk University, Irbid, Jordan

E-mail: anagrehy@yu.edu.jo

**Abstract** This paper presents an attractive approach for teaching the self-excited induction generator. Three operating conditions of the generator are mathematically modeled and then simulated using conventional Matlab commands. Active windows with these models are created using Matlab's Graphical User Interface capability. An example is given to demonstrate the usefulness of the developed tool.

**Keywords** computer simulations; induction generators; teaching aids

## List of symbols

$R_S, R_R, R_M, R_L, R_C$	p.u stator, rotor, magnetizing, load and exciting resistances, respectively
$X_S, X_R, X_M, X_L, X_C$	p.u stator, rotor leakage, magnetizing, load and exciting reactances at base frequency, respectively
$Y_S, Y_R, Y_M, Y_L, Y_C$	p.u stator, rotor, magnetizing, load and exciting admittances, respectively
$f_s$	synchronous frequency
$F$	p.u frequency
$\nu$	p.u rotational speed
$E_g, V_T$	p.u air gap and terminal voltages, respectively
$I_S, I_R, I_L$	pu stator, rotor and load currents per phase, respectively
$P_{out}$	output electrical power
$P_{in}$	input mechanical power
$Z_C$	capacitor bank impedance

Three-phase squirrel cage induction generators are usually implemented in stand-alone power systems that employ renewable energy resources, like hydro-power and wind energy.<sup>1-4</sup> This is due to the advantages of these generators over conventional synchronous generators. The main advantages are: reduced unit cost, absence of a separate d.c. source for excitation, ruggedness, brushless rotor construction and ease of maintenance.

This paper presents an effective approach for teaching the self-excited induction generator (SEIG) using a Matlab environment. This approach enables a student to simulate different operating conditions of the generator easily using basic Matlab instructions. In addition, the student can learn Matlab's Graphical User Interface (GUI) capabilities to develop an attractive and user-friendly software package for teaching a SEIG.

The next section presents not only the requirements to operate an induction

machine as a SEIG, but also the parameters affecting the performance of this generator. Mathematical modeling and simulations of a SEIG under three operating conditions (constant frequency operation, constant speed operation and variable frequency-variable speed operation) are then given. As a basis for developing an attractive and easy-to-use educational software tool, some of Matlab's GUI functions are implemented in creating an active link with these models. The next section demonstrates the effectiveness of the presented approach using an example of a SEIG under variable speed-variable frequency operating condition. Finally, conclusions of this work are given.

### Self-excited induction generator

A three-phase induction machine can be operated as a SEIG if its rotor is externally driven at a suitable speed and a three-phase capacitor bank of a sufficient value is connected across its stator terminals. The block schematic diagram of a SEIG system is shown in Fig. 1. When the induction machine is driven at the required speed, the residual magnetic flux in the rotor will induce a small e.m.f. in the stator winding. The appropriate capacitor bank causes this induced voltage to continue to increase until an equilibrium state is attained due to magnetic saturation of the machine.

Fig. 2 shows both the magnetizing curve of an unloaded SEIG and the voltage-current characteristic of a capacitor bank plotted on the same set of axes. The intersection of the two curves is the point at which the capacitor bank exactly supplies the reactive power demanded by the generator. As shown in the figure the no-load terminal voltage of the generator may be determined from this point.

When a SEIG is loaded, both the magnitude and frequency of the induced e.m.f. are affected by: the prime mover speed, the capacitance of the capacitor bank and the load impedance. To simplify the discussion, we assume that all losses in the generator are ignored ( $P_{in} = P_{out}$ ), the connected load is purely resistive and the rotor speed is kept constant. In this case, the decrease in the load resistance (increase in  $P_{out}$ ) will result in a drop in the stator frequency to provide higher torque to match the increment in the power demand. This circumstance is illustrated in Figs 3 and 4.

In the above case, it is assumed that the SEIG was feeding a resistive load so that

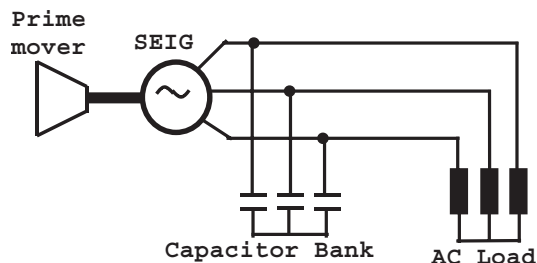


Fig. 1 Self excited induction generator system.

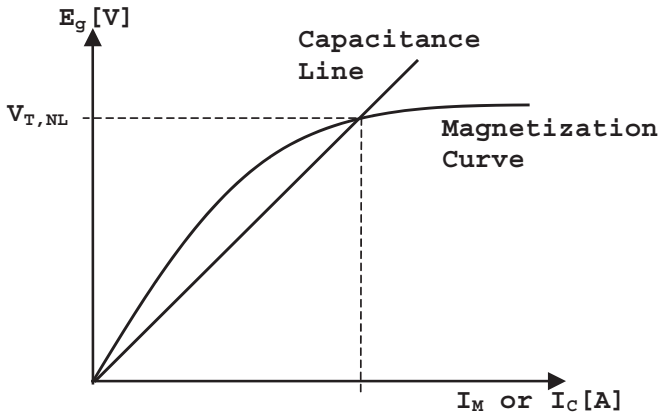


Fig. 2 No-load magnetization characteristic and capacitive impedance line.

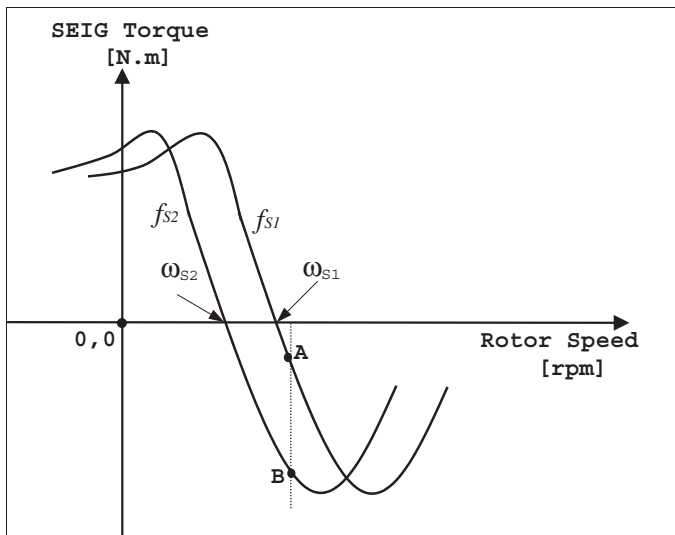


Fig. 3 Torque speed characteristics of an induction machine,  $f_{s1} > f_{s2}$ .

the steady-state operation point was A and the synchronous frequency was  $f_{s1}$ . When the power demand increases, the synchronous frequency reduces from  $f_{s1}$  to  $f_{s2}$  and the new steady-state operation point is changed to point B. The reduction in the synchronous frequency causes the induced e.m.f. to reduce in the same proportion, if  $\text{emf}/f_s$  ratio is kept constant. Moreover, the decrease in  $f_s$  changes the slope of the impedance line from  $Z_{C1}$  to  $Z_{C2}$ , as shown in Fig. 4, according to the following relationship:

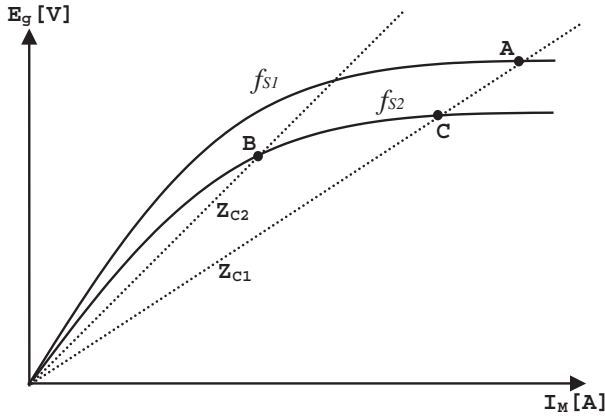


Fig. 4 Magnetization characteristics and capacitive impedance lines of a loaded SEIG,  $f_{s1} > f_{s2}$ .

$$Z_c = \frac{1}{2\pi f_s C}$$

The induced e.m.f. could be increased, by returning the slope of the  $Z_c$  line back to its previous value (increasing the capacitance value of the capacitor bank). Since the synchronous frequency remains  $f_{s2}$ , the steady-state operation point will now be at point C instead of point A.

**Modeling and simulation of SEIG**

The steady-state per-phase equivalent circuit of a SEIG, supplying a balanced resistive load, is shown in Fig. 5. In this circuit, only the magnetizing reactance is assumed to be affected by magnetic saturation, and all other parameters are assumed to be constant. In addition, core losses and the effect of the harmonics are ignored. From Fig. 5, the total current at node a may be given by:<sup>5,6</sup>

$$E_1(Y_1 + Y_M + Y_R) = 0 \tag{1}$$

Therefore, under steady-state self-excitation, the total admittance must be zero, since  $E_1 \neq 0$ .

$$Y_1 + Y_M + Y_R = 0 \tag{2}$$

or

$$Real(Y_1 + Y_M + Y_R) = 0 \tag{3}$$

$$Imag(Y_1 + Y_M + Y_R) = 0 \tag{4}$$

Where

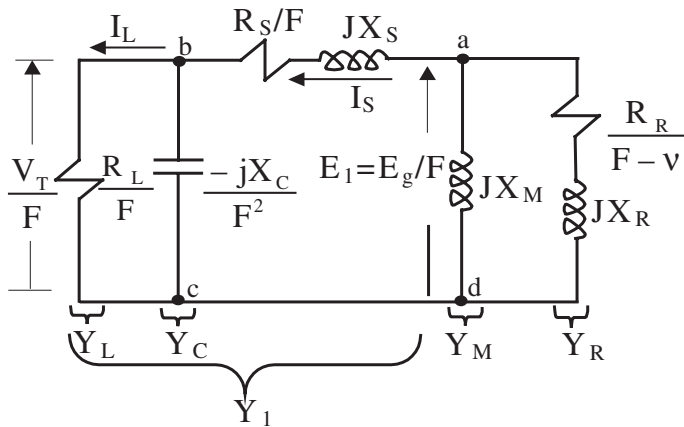


Fig. 5 Per-phase equivalent circuit of a SEIG.

$$\begin{aligned}
 Y_1 &= \frac{(Y_C + Y_L)(Y_S)}{Y_C + Y_L + Y_S} & Y_C &= \frac{1}{-(jX_C/F^2)} & Y_L &= \frac{1}{(R_L/F)} \\
 Y_S &= \frac{1}{(R_S/F) + jX_S} & Y_M &= \frac{1}{jX_M} & Y_R &= \frac{1}{\frac{R_R}{F-v} + jX_R}
 \end{aligned}$$

Equations 3 and 4 are nonlinear equations of four unknowns:  $F$ ,  $X_M$ ,  $X_C$  and  $v$ . Two of these unknowns should be specified. The other two unknowns can be found by solving eqn. 3 for one unknown and the other unknown is calculated from eqn. 4. Having determined the two unknown parameters, the steady-state performance of the SEIG can be obtained by solving the circuit of Fig. 5 with the help of the generator's magnetization curve. The performance characteristics of the generator could be estimated using the following relationships:

$$\begin{aligned}
 I_S &= \frac{E_g/F}{\frac{R_S}{F} + jX_S - \frac{jX_C R_L}{F^2 R_L - F X_C}} & I_R &= \frac{-E_g/F}{\frac{R_R}{F-v} + X_R} & I_L &= \frac{-jX_C I_S}{R_L F - jX_C} \\
 V_T &= I_L R_L & P_{in} &= \frac{-3R_R F |I_R|^2}{F-v} & P_{out} &= 3|I_L|^2 R_L
 \end{aligned}$$

Three operating conditions of a SEIG are considered in the present work. These are constant speed operation, constant frequency operation and variable frequency-variable speed operation. To achieve constant speed operation, the generator should be driven by a fixed shaft speed turbine. To maintain the frequency of the output voltage constant, a controlled shaft speed turbine may be implemented to drive the generator. If an unregulated turbine drives the generator, the frequency and shaft speed are affected by the energy source. For example, these two parameters are

affected by wind speed if the generator's prime mover is an unregulated wind turbine.

Based on eqns 3 and 4, the above three operating conditions are mathematically modeled. If the generator is operated at constant shaft speed,  $F$  and  $X_M$  will vary with the load and hence they must be taken as the two unknowns. For this case, a fifth order polynomial, independent of  $X_M$ , is extracted from eqn. 3, and the  $X_M$  eqn. is obtained from eqn. 4 as follows:

$$A_5 F^5 + A_4 F^4 + A_3 F^3 + A_2 F^2 + A_1 F + A_0 = 0 \quad (5)$$

$$X_M = -1 / \left\{ \frac{X_R}{[R_R / (F - \nu)]^2 + X_R^2} + \frac{X_{ac}}{R_{ac}^2 + X_{ac}^2} \right\} \quad (6)$$

where

$$A_0 = -\nu R_R \left( \frac{R_3}{R_L} \right)^2 \quad A_1 = R_R \left( \frac{R_3}{R_L} \right)^2 + R_3 \left( \frac{R_R}{R_L} \right)^2 + \nu^2 R_3 \left( \frac{X_R}{R_L} \right)^2$$

$$A_2 = -2\nu R_3 \left( \frac{X_R}{R_L} \right)^2 - \nu R_R \left\{ \left( \frac{R_S}{X_C} \right)^2 + \left( \frac{X_S}{R_L} \right)^2 - 2 \left( \frac{X_S}{X_C} \right) \right\}$$

$$A_3 = R_R \left[ \left( \frac{X_S}{R_L} \right)^2 + \left( \frac{R_S}{X_C} \right)^2 - 2 \left( \frac{X_S}{X_C} \right) \right] + R_3 \left( \frac{X_R}{R_L} \right)^2 + R_3 \left( \frac{R_R}{X_C} \right)^2 + \nu^2 R_3 \left( \frac{X_R}{X_C} \right)^2$$

$$A_4 = -\nu \left[ R_R \left( \frac{X_S}{X_C} \right)^2 + 2R_S \left( \frac{X_R}{X_C} \right)^2 \right] \quad A_5 = R_R \left( \frac{X_S}{X_C} \right)^2 + R_S \left( \frac{X_R}{X_C} \right)^2$$

$$R_{bc} = R_L X_C^2 / [F(F^2 R_L^2 + X_C^2)] \quad X_{bc} = R_L^2 X_C / (F^2 R_L^2 + X_C^2)$$

$$R_{ac} = \frac{R_S}{F} + R_{bc} \quad X_{ac} = X_S - X_{bc} \quad R_3 = R_S + R_L$$

When a SEIG is operated at constant frequency,  $\nu$  and  $X_M$  should be considered as unknowns. For this operating condition, eqn. 3 can be simplified to give eqn. 7, which is a quadratic equation of  $\nu$ . For a given  $F$ ,  $X_C$  and  $R_L$ , eqn. 7 can be used to obtain  $\nu$  and then  $X_M$  may be calculated from eqn. 6.

$$B_2 \nu^2 + B_1 \nu + B_0 = 0 \quad (7)$$

where

$$B_0 = R_R F (R_{ac}^2 + X_{ac}^2) + R_{ac} (R_R^2 + F^2 X_R^2) \quad B_1 = -[R_R (R_{ac}^2 + X_{ac}^2) + 2F R_{ac} X_R^2]$$

$$B_2 = R_{ac} X_R^2$$

For the variable frequency-variable speed operating condition,  $F$  and  $X_M$  are affected by both  $R_L$  and  $\nu$ . Therefore,  $F$  and  $X_M$  can be considered as the unknown param-

ters, but the effects of both  $R_L$  and  $\nu$  on these unknowns should be considered. For a given  $\nu$ ,  $X_C$  and  $R_L$ , eqn. 5 can be solved to determine  $F$ , and  $X_M$  can be computed using eqn. 6.

The mathematical models representing the three operating conditions of a SEIG could be simulated in separated script files (M-files) using conventional Matlab language. The created Matlab code to simulate the variable frequency-variable speed operating condition is:

```
%The parameters of the per-phase equivalent circuit of
%the SEIG in PU
Rs=0.1; Xs=0.2; Rr=0.06; Xr=0.2; Xc=1.2; vv=0.7; FF=0.4;
%The speed and frequency
N=0.9; Freq=0.9;
%The increment, and the last values for speed and RL
%counters,
zz=10; kk=4; inc=0.1;

%Variable frequency-variable speed operating condition
if (m2==1) default zz=50;
else user end
for ii=1:kk;
vv=vv+inc;
RLi=1.2;
    for jj=1:zz;
        RLi=RLi+.6;
        RL(jj,ii)=RLi;

R3=Rs+RL(jj,ii);
A5=Rr*(Xs/Xc)*(Xs/Xc)+Rs*(Xr/Xc)*(Xr/Xc);
A4=-vv*(Rr*(Xs/Xc)*(Xs/Xc)+2*Rs*(Xr/Xc)*(Xr/Xc));
A3_a=Rr*((Xs/RL(jj,ii))*(Xs/RL(jj,ii))+(Rs/Xc)*(Rs/Xc)
-2*(Xs/Xc));
A3_b=R3*(Xr/RL(jj,ii))*(Xr/RL(jj,ii))
+Rs*(Rr/Xc)*(Rr/Xc);
A3_c=vv*vv*Rs*(Xr/Xc)*(Xr/Xc); A3=A3_a+A3_b+A3_c;
A2_a=-2*vv*R3*(Xr/RL(jj,ii))*(Xr/RL(jj,ii));
A2_b=-vv*Rr*((Rs/Xc)*(Rs/Xc)+
(Xs/RL(jj,ii))*(Xs/RL(jj,ii))-2*(Xs/Xc));
A2=A2_a+A2_b;
A1=Rr*(R3/RL(jj,ii))*(R3/RL(jj,ii))
+R3*(Rr/RL(jj,ii))*(Rr/RL(jj,ii))+vv*vv*R3*...
(Xr/RL(jj,ii))*(Xr/RL(jj,ii));
A0=-vv*Rr*(R3/RL(jj,ii))*(R3/RL(jj,ii));
coeff=[A5 A4 A3 A2 A1 A0];
R=[roots(coeff)]; R=R';
```

```

M(jj,1:5)=[R]; M1=M(jj,1); M2=M(jj,2); M3=M(jj,3);
M4=M(jj,4); M5=M(jj,5);

%Check if the roots are real or complex.
C1=isreal(M1); C2=isreal(M2); C3=isreal(M3);
C4=isreal(M4); C5=isreal(M5);

%check if a root is real & positive and only the
%positive real root is taken
if C1==1 & M1>0 & M1<1; F(jj,ii)=M1;
elseif C2==1 & M2>0 & M2<1; F(jj,ii)=M2;
elseif C3==1 & M3>0 & M3<1; F(jj,ii)=M3;
elseif C4==1 & M4>0 & M4<1; F(jj,ii)=M4;
elseif C5==1 & M5>0 & M5<1 F(jj,ii)=M5;
else F(jj,ii)=1;
end

%Obtaining the value of the magnetizing reactance Xm
FV(jj,ii)=F(jj,ii)-vv; Zp1(jj,ii)=RL(jj,ii)/F(jj,ii);
Zp2(jj,ii)=(-Xc*i)/(F(jj,ii)*F(jj,ii));
Zp(jj,ii)=(Zp1(jj,ii)*Zp2(jj,ii))/
(Zp1(jj,ii)+Zp2(jj,ii));
Zs(jj,ii)=(Rs/F(jj,ii))+(Xs*i);
Zps(jj,ii)=Zp(jj,ii)+Zs(jj,ii);
Yps(jj,ii)=1/Zps(jj,ii);
Yr(jj,ii)=1/((Rr/FV(jj,ii))+(Xr*i));
Yps_R(jj,ii)=imag(Yps(jj,ii));
Yr_R(jj,ii)=imag(Yr(jj,ii));
Ym(jj,ii)=- (Yr_R(jj,ii)+Yps_R(jj,ii));
Xm(jj,ii)=-1/Ym(jj,ii);

```

The 'User' can access Matlab's GUI facilities to develop a powerful and attractive software package for teaching a SEIG. As an example of using Matlab's GUI capabilities, menu and plotting commands are implemented in a script file to provide interactive windows with the three simulated operating conditions. The main menu, which is displayed after running the file, is shown in Fig. 6.

### Demonstration

The ease of using the presented tool for accurate analysis of a SEIG has been demonstrated by examining the variations of  $V_T$  with  $I_L$  at different prime mover speeds. The SEIG chosen for this study is a 3-phase, 4-pole, 60Hz, 1kW, 380V, 2.27A, Y-connected squirrel cage induction machine whose per phase equivalent circuit parameters in pu are:

$$R_s = 0.1 \quad X_s = 0.2 \quad R_r = 0.06 \quad X_r = 0.2.$$

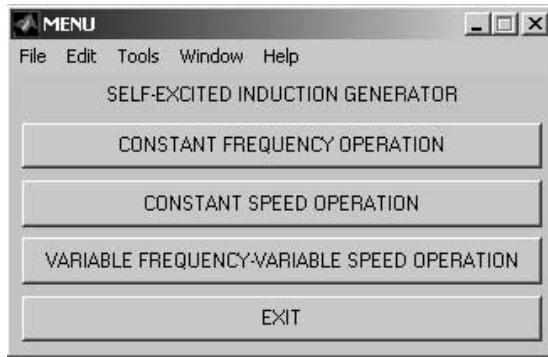


Fig. 6 The main window of the developed software tool.

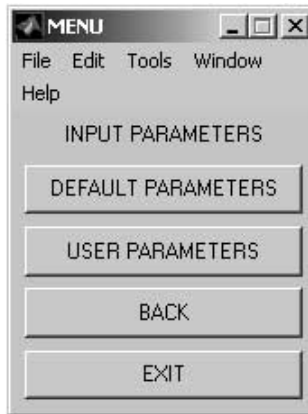


Fig. 7 The window of inserting the inputs for the desired SEIG.

The relationship between the magnetizing reactance  $X_m$  and the air-gap voltage  $E_g/F$  of the machine is:

$$\frac{E_g}{F} = 1.12 + 0.078X_M - 0.146X_M^2 \quad 0 < X_M < 3$$

This study is carried out using the created GUI windows. Running the created GUI M-file, called *main\_menu*, from Matlab workspace will display the main window shown in Fig. 6. The window shown in Fig. 7 will appear after clicking on the icon named 'VARIABLE FREQUENCY\_VARIABLE SPEED OPERATION'. Selecting 'DEFAULT PARAMETERS' option will present the result menu shown in Fig. 8. The terminal voltage-load current characteristics can be displayed on the graph window by just clicking on 'VT-IL' icon. The  $V_T-I_L$  characteristics, shown in Fig. 9, are in full agreement with those obtained, by other authors.<sup>1,6</sup> It can be seen that the



Fig. 8 The window of the computed results for the desired SEIG.

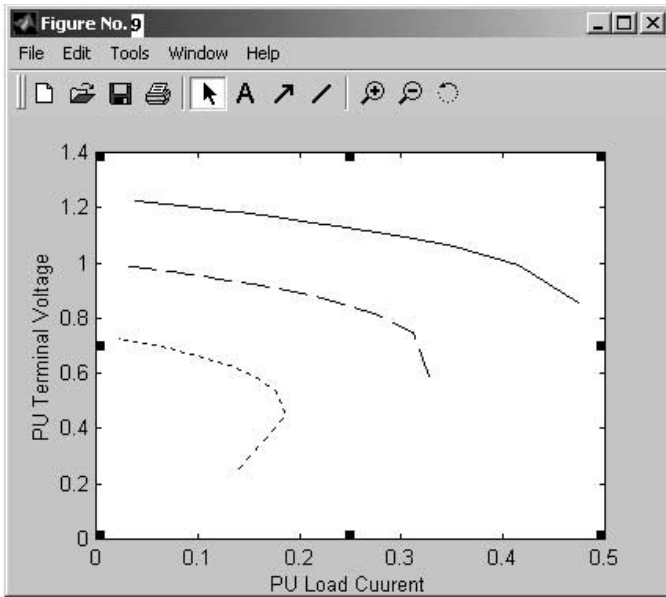


Fig. 9  $V_T-I_L$  characteristics under variable frequency-variable speed operating condition  
.....  $\nu = 0.8 pu$ ,      - - - -  $\nu = 0.9 pu$ ,      —  $\nu = 1 pu$ .

terminal voltage drops as the load current increases and it increases approximately linearly with prime mover speed.

## Conclusion

An attractive and useful approach for teaching a SEIG has been presented in this article. Three operating conditions of the generator are mathematically modeled then simulated using basic Matlab instructions. The developed tool is made easy to use by providing an active link with the simulated models using some of Matlab's GUI functions. The given example demonstrates the usefulness of the developed tool for teaching self-excited induction generators.

## References

- 1 S. S. Murthy, O. P. Malik and A. K. Tandon, 'Analysis of self-excited induction generators', *IEE Proc. C.*, **129** (5) (1982), 260–265.
- 2 N. Ammasaigounden, M. Subbiah and M. R. Krishnamurthy, 'Wind-driven self-excited pole-changing induction generators', *IEE Proc. B.*, **133** (5) (1986), 315–321.
- 3 G. Grantham, D. Sutanto and B. Mismail, 'Steady-state and transient analysis of self-excited induction generators', *IEE Proc. B.*, **136** (2) (1989), 61–68.
- 4 T. F. Chan, 'Self-excited induction generators driven by regulated and unregulated turbines', *IEEE Trans.*, **EC-11** (2) (1996), 338–343.
- 5 S. M. Alghuwainem, 'Performance analysis of a PV powered DC motor driving a three phase self-excited induction generator', *IEEE Trans.*, **EC-11** (1) (1996), 155–161.
- 6 L. A. Alolah and M. A. Alkanhal, 'Optimization-based steady state analysis of three phase self-excited induction generator', *IEEE Trans.*, **EC-15** (1) (2000), 61–65.