An efficient power flow algorithm for distribution systems with polynomial load

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Abstract A new, efficient power flow algorithm for complex distribution systems is presented. Voltage ratio is used for convergence control. This method has fast convergence ability for the polynomial load model for which the traditional Newton-Raphson method is usually not adaptable. Test results show the robustness of the proposed method.

Keywords Newton-Raphson method; polynomial load model; power flow; radial network

Power flow analysis is essential for power system planning and operation. With the use of digital computing since the 1960s and its rapid development, many power flow algorithms based on modern computing methods have been introduced.[1–3] The most famous methods include: ladder network methods for radial-type distribution systems using basic circuit theories (KCL and KVL); and the Gauss-Seidel, Newton-Raphson and Decoupled Newton-Raphson methods for transmission grid analysis using the nodal method. All these methods have been successfully applied in industry for many years. Well-known conclusions of using these methods are:[3–6]

- ladder network methods are quite suitable for one sending end radial networks with high R/X ratio;
- nodal analysis methods are suitable for multiple-source systems.

Traditionally, most distribution systems are radial or weakly meshed types.[4–7] Faced with the power markets of today, increasing requirements for reliability and outgoing distribution generation have meant that the structure of distribution systems has become more complex.[1,8] Thus the power flow analysis in such distribution systems becomes more difficult than before.

Most of the above methods were developed based on the assumptions of a static load model. Reliable power delivery needs power flow analysis with a detailed load model. Distributed generation is becoming an attractive solution to meet the fast load increase in the deregulation era.[9] Utilities have to analyse the operation conditions of the radial-type systems with distributed sources. Newton-Raphson like methods are not suitable for this purpose because of the high R/X ratio; while traditional ladder network methods also face a great challenge because of the multiple-source conditions. Usually the commercial SCADA/DMS systems treat these systems as independent parts, i.e., HVAC (high voltage a.c.) loop and MVAC (medium voltage a.c.) or LVAC (low voltage a.c.) radial systems. Such rough equivalence will cause inaccuracies in the power flow solutions.

The objective of this paper is to find a robust power flow algorithm that can handle power flow problems in modern complex distribution systems, which have multiple sources or strong connected loops with long radial networks. Since the Newton-Raphson method has been well acknowledged for its robustness in loop-type systems, the above problem is solved in two steps:

- to develop a robust power flow algorithm for radial systems. It must be at least as robust as the Newton-Raphson method but more efficient for the networks with high R/X ratio;
- to combine the radial and the Newton-Raphson algorithm together to solve the whole system with multiple sources.

Literature review of distribution power flow algorithms

Distribution systems usually fall into the category of ill-conditioned power systems for generic Newton-Raphson like methods with its special features,[1] such as radial or weakly meshed topologies, high R/X ratio of the distribution lines, unbalanced operation and loading conditions, non-linear load models and dispersed generation, etc. Numerous efforts have been made to develop power flow algorithms for distribution systems. The following are the most typical ones:

Forward and backward sweep methods or ladder networks theory

These methods take advantage of a natural feature of the radial networks, i.e., there is a unique path from any given bus to the source. The general algorithm consists of two basic steps: forward sweep and backward sweep.[1,4,6] Salama *et al.* have presented a very simple but robust method – the ladder formula.[4] Essentially, the ladder network method treats the radial system as two basic element types: the network natural elements (impedance) and voltage control current sources (system loads) at each load node. The forward sweep is mainly a voltage drop calculation from the sending end to the far end of a feeder or a lateral; and the backward sweep is primarily a current summation based on the voltage updates from the far end of the feeder to the sending end. Then by using KVL and KCL, the voltage drop can be obtained.[6]

Berg *et al.* presented a backward method in 1967,[10] which used a backward procedure to update the equivalent impedance at the sending end. The main idea of this method is to treat the load as constant impedance. So if the equivalent impedance is convergent, the whole system convergence will be reached. This method is very costly and quite sensitive to the system load level and load distribution, as well as the system structures.

Baran *et al.* presented a forward method in 1989.[11] In this method, the sending end voltage becomes the main concern of the system convergence. Voltage drop and the information on system structure have been considered in the forward sweep. The voltage-sensitive load current can be included in the system model. However, this method still has disadvantages. Oriented from ladder network concepts, the ‘branch flow equations' are essentially solved by a Newton-Raphson approach which makes this method complex and costly. Sometimes the influence of the load distribution would cause slow convergence.

Most recently, Nanda *et al.* presented a new backward algorithm.[8] Since the heavy load in the far end of the feeder usually causes more iteration, this method uses the backward sweep to calculate the sending end voltage. Although the heavy load problem could be solved, the disadvantage of this method is the difficulty of the convergence – because the backward calculation of the voltage drop usually causes the voltage of the sending end to go very high compared to the setting value. So a special value is needed to control the voltage profile during iterations in terms of the convergence tolerance. The authors did not explain how to determine these special values, theoretically or heuristically.

In 1995, Shrimohammadi *et al.* stated that the ladder network method could be performed in two directions: a backward sweep for current summation and then a forward voltage calculation.[5,12] This iteration sequence could speed up the convergence compared to the forward ladder network method.

Generally, the above ladder network methods have the following advantages:

- quite robust for heavy loads;
- less sensitive to the high R/X ratio;
- simple formulation;
- more suitable to reflect the dependency of the node voltage on the load level, which is a distinguishing characteristic of distribution systems.

The limitation is that these methods are only suitable for one source and a simple tree structure. Load distribution and tree structure still influence the convergence speed.

Compensation-based open loop distribution power flow for weakly meshed networks

Some distribution systems have weakly meshed structures. Because of the interconnection of the system branches, the methods mentioned in the previous section would not be suitable. Because of the high R/X ratio, Newton-Raphson like methods would not be adaptable either.

In order to solve such problems, Luo *et al.* presented a series of papers[5,7] on a compensation method for weakly meshed networks. This method started from a network structure analysis to find the interconnection points. Then it breaks these interconnection points using the compensation method so that the meshed system structure could be changed to simple tree-type radial system. This method is also suitable for the system with multiple voltage control buses.

Haque presented a new approach for meshed networks with more than one feeding node.[12] The method first converts the multiple-source mesh network into an equivalent single-source radial type network by setting dummy nodes for the break points at distributed generators and loop connecting points. Then the traditional ladder network method can be applied for the equivalent radial system. Following each of the iterations of the equivalent radial system, the power injected at the breaker points must be updated by an additional calculation through a reduced order impedance matrix.

The disadvantages of the above methods are:

- the structure analysis of the system is complex. Sometimes a heuristic method must be used to decide where the break points are. So the adaptability of this method is not good enough.
- load conditions at the break points have great influence on the power flow solutions. Heavy loading loops and weak power sources (dispersed generation) in meshed systems may cause difficulties for the convergence speed and accuracy.

Using current injection in Newton-Raphson and Newton-Raphson like methods to solve power flow problems for large distribution systems

As discussed earlier, with the expansion of distribution systems strong connection loops exist in modern distribution networks. When using the Newton-Raphson method for such networks, how to obtain the equivalent network of the rest of the system becomes very important. In distribution systems, equivalent impedance methods, which are popular in transmission systems, are no longer suitable because of the load behaviours. Then load current injection becomes a good choice.[2,13] This method is still based on the nodal model. The formulas are complex, and the computation cost is high as well, especially when the system load model is voltage sensitive.

Distribution load modelling

In distribution systems, because of the voltage-dependent characteristics of load, the constant load model is no longer suitable for accurate power flow analysis.[14,15] Load models usually can be classified into two main categories: static and dynamic. Since power flow analysis is mainly performed for static states of power systems, only static load is considered here.

Normally, static load can be described using one of the following models:

- constant impedance model (constant Z), i.e., the load power varies with the square of the voltage magnitude;
- constant current model (constant I), i.e., the load power varies with the voltage magnitude only;
- constant power model (constant P and Q), i.e., the load power doesn't vary with the voltage magnitude;
- exponential load model, i.e., the load power varies with the voltage magnitude with an exponential relationship.

In this paper, the load is modelled as polynomial load[4,6] as:

$$P = P_0\left(a_0 + a_1 V + a_2 V^2 + a_3 V^{1.38}\right)$$
$$Q = Q_0\left(b_0 + b_1 V + b_2 V^2 + b_3 V^{3.22}\right) \qquad (1)$$
$$a_0 + a_1 + a_2 + a_3 = b_0 + b_1 + b_2 + b_3 = 1$$

where

V is the p.u. value of the node voltage;

P_0,Q_0 are the real power and reactive power consumed at the specific node under the reference voltage;

a_0,b_0 are the parameters for constant power (constant P and Q) load component;

a_1,b_1 are the parameters for constant current (constant I) load component;

a_2,b_2 are the parameters for constant impedance (constant Z) load component;

a_3,b_3 are the parameters for exponential load component.

The values of a_0,b_0, a_1,b_1, a_2,b_2, a_3,b_3 are determined for different load types in distribution systems. Usually experimental or experience values could be used. The exponential load parameters in formula (1) come from Ontario Hydro.

Proposed algorithm for radial network power flow: ratio-flow

The first approach of this paper is to develop a power flow method for the radial part of a complex distribution system. The basic requirements of this method are:

- low sensitivity to high R/X ratio networks;
- fast convergence ability, i.e., the convergence speed should not be sensitive to the feeder lengths and the number of lateral branches.

For the first requirement, the ladder network method is chosen. The study is then concentrated on how to improve the convergence speed of the ladder network method. A typical radial distribution system is shown in Fig. 1. For balanced power flow analysis, a ladder network method is based upon the per phase circuit in Fig. 2.

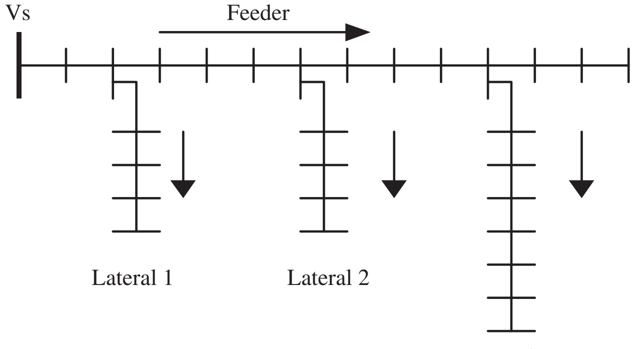

Fig. 1 *A typical radial-type distribution system.*

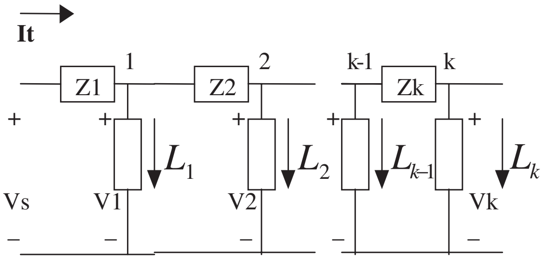

Fig. 2 *The equivalent circuit for the ladder network method.*

The forward ladder formula is:

$$V_n = V_s - I_t \sum_{k=1}^{n} Z_k + \sum_{k=1}^{n-1} I_k \left(\sum_{i=k+1}^{n} Z_i \right) \tag{2}$$

and the backward ladder formula is

$$V_s = V_n + I_t \sum_{k=1}^{n} Z_k - \sum_{k=1}^{n-1} I_k \left(\sum_{i=k+1}^{n} Z_i \right) \tag{3}$$

where

n is the number of nodes in the feeder;
V_s,V_n stand for the sending end voltage and the far end voltage respectively;
Z_k is the impedance of the *kth* section of the feeder;
L_k is the load at node k;
I_k is the load current at node k;
I_t is the total current sent from the sending end node.

From the above two formulas, the following conclusions can be drawn:

- using the ladder network method, the computation cost mainly relies on the number of nodes in the radial tree;
- both the network characteristics (Z_k) and the load behaviour (I_k) are included in the voltage profile. Thus the accuracy of the nodal voltages becomes the key point.

Since nodal voltages are controlled by the sending end voltage (V_s) and affected by load currents of radial type distribution networks, the following work concentrates on how to improve the accuracy of load currents and voltage profile at each iteration loop so that high accuracy and fast speed can be expected. The proposed method will be developed gradually from a simple system (one feeder without sub-branch) to general cases (feeder with multiple lateral branches).

One feeder without sub-branch (lateral)

As discussed above, the forward sweep method is based on the natural voltage distribution, i.e., all the node voltages are controlled by V_s; while the backward sweep

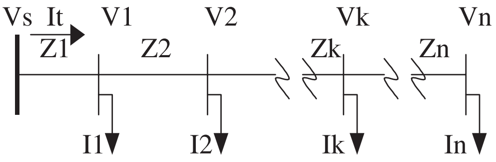

Fig. 3 *One feeder without sub-branch (lateral).*

method concentrates on the influences of the load current on the node voltage distribution. The forward-backward method combines the two methods. Analysis of these three methods shows that the forward method has a fast convergence characteristic. The backward method sometimes needs more iteration for long feeders, and the initial value is very important.

The forward-backward method is better than the other two methods. However, when the feeder is longer, the convergence speed becomes quite slow. Then the goal is to improve the convergence speed of the forward-backward method.

Consider the simple network in Fig. 3, one feeder without sub-branch; the convergence criteria are chosen as the difference of the voltage profiles between the two adjacent iterations, j and $j + 1$, i.e.,

$$\left\| \left| V_k^{j+1} \right| - \left| V_k^j \right| \right\| < \varepsilon \tag{4}$$

If the voltage profile can be properly chosen so that all the node voltage drops can be revised reasonably after the backward sweep in each iteration, the accuracy of the current profile at the forward sweep will be improved. Then the overall backward-forward iteration will be speeded up. This idea is implemented in the proposed algorithm, ratio-flow, which consists of the following procedures for one feeder without sub-branch:

Step 1: Initialise the iteration counter, i.e., $i = 1$; initialise the voltage values of all feeder buses, i.e., $v_1 = v_2 = \ldots = v_n = 1$ p.u., V_s is usually chosen as $V_s = 1$ p.u.

Step 2: Calculate the load current profile:

$$I_K = \left(\frac{S_K}{V_K} \right)^* \tag{5}$$

with the node voltage profile V_K and the polynomial load model (formula (1)).

Step 3: Perform the backward sweep using formula (3) to obtain a new voltage profile V_K^{New} and new sending end voltage V_S^{New}.

Step 4: Calculate the ratio of the node voltage to the new sending end voltage, which is defined as:

$$V_K\_Ratio = \frac{V_K^{New}}{V_S^{New}} \tag{6}$$

Calculate the ratio of the new sending end voltage to the given value, which is defined as:

$$V_S\_Ratio = \frac{V_S^{New}}{V_S} \tag{7}$$

Step 5: Adjust the V_K^{New} as

$$V_K^{Adjust} = \frac{V_K^{New}}{V_S\_Ratio} \tag{8}$$

Step 6: Recalculate the load current profile using

$$I_K = \left(\frac{S_K^{Adjust}}{V_K^{Adjust}} \right)^{*} \tag{9}$$

where S_K^{Adjust} is the adjusted polynomial load profile obtained from formula (1) with V_K^{Adjust}.

Step 7: Perform the forward sweep using formula (2) with the given V_S and the new current profile obtained from *Step 6* to calculate the desired feeder node voltages V_K.

Step 8: Return to *Step 2* until the convergence tolerance (formula (4)) is reached.

Test results show that ratio-flow provides robust solutions for different lengths of the radial feeders and it is more suitable for a polynomial load compared with the forward method.

Another interesting result is that the ratio of the node voltage to the new sending end voltage V_K_Ratio, which is obtained from *Step 6*, has a very similar convergence speed to V_K. It is a very useful conclusion that will be used as the local convergence criteria for the multiple branch radial systems.

General case: feeder with multiple lateral branches

The ladder network methods, introduced in Refs,[4,7,8] used the same convergence strategy, i.e., checking the whole system voltage convergence after each overall iteration. Thus, if one of the lateral branches needs more iteration, the whole system convergence speed will be slowed down. Considering the voltage drop characteristic of the single source radial type distribution systems, this work uses a local convergence technique to reduce the overall system calculation cost.

The whole system power flow analysis is divided into two sub-iterations (Fig. 4).

Lateral sub-iteration

For each lateral, the voltage-ratio V_K_Ratio is used as the lateral convergence target rather than the node voltage itself during the above speed-up forward-backward process. When the lateral reaches 'lateral local convergence', the lateral total load current is calculated and added to its sending end on the feeder.

Feeder sub-iteration

The feeder sub-iteration performs the above speed-up forward-backward process until the voltages along the feeder reach the 'feeder local convergence' based on the current injections from *Lateral sub-iteration.*

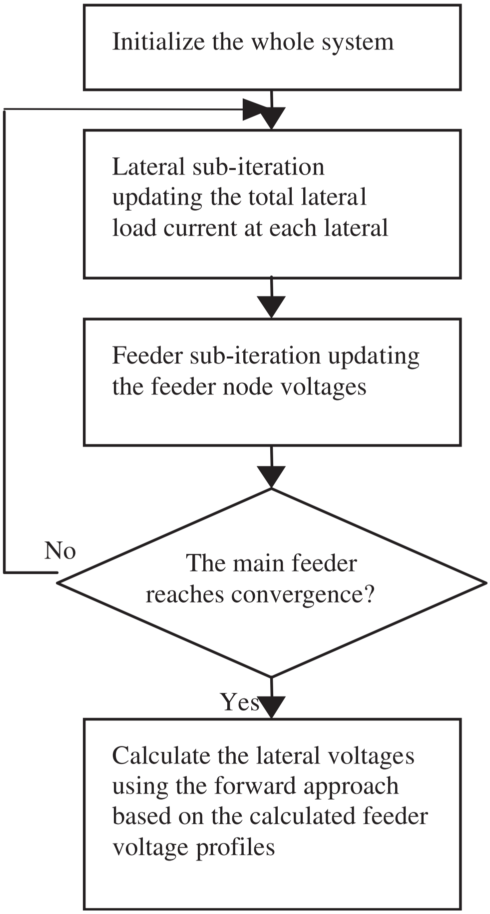

Fig. 4 *The flowchart of ratio-flow for one feeder with sub-branches.*

The main iteration will converge when the two adjacent feeder sub-iterations reach a tolerable accuracy, i.e., the main iteration tolerance. Then an additional forward approach using formula (2) will be used to calculate the lateral branch voltage profiles based on the feeder voltage distribution.

Proposed algorithm for complex distribution systems: hybrid distri-flow

When there exists more than one source in the radial networks or the radial networks are connected to a strong connected loop, the sending end cannot be equivalent to an infinite system bus. Then the ladder network method in its present form will not be suitable any more. In order to solve the above problems, a hybrid distribution power flow algorithm, hybrid distri-flow, a combination of the Newton-Raphson method and the proposed ratio-flow method, is developed in this work.

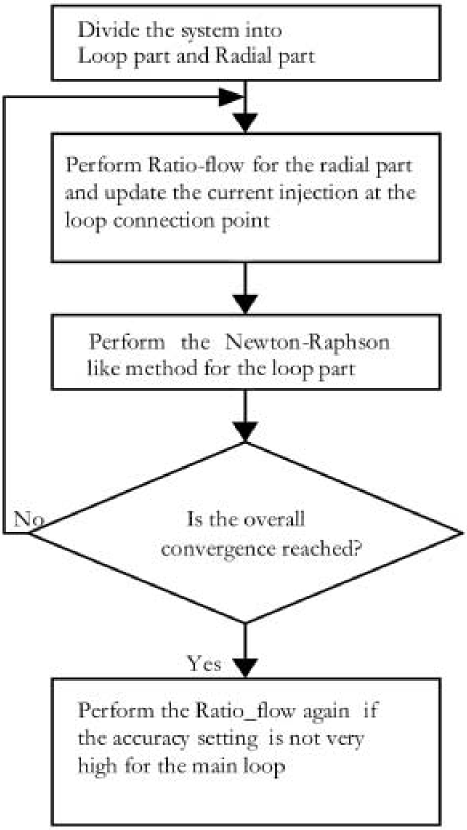

Fig. 5 *The flowchart of hybrid distri-flow.*

The basic idea of this algorithm is the superposition procedure (Fig. 5).

Step 1: *Divide the whole system into two main parts:* small loops which contain the independent sources and radial parts.

Step 2: *Radial sub-iteration*:

- Apply the ratio-flow algorithm to calculate the power flow for the radial part;
- Update the total radial current until the radial part reaches 'radial local convergence'.

Step 3: *Loop* sub*-iteration:*

- Use a standard Newton-Raphson algorithm to calculate the power flow for the loop part based on the updated radial current injection obtained from *Step 2*, until the loop part reaches 'loop local convergence'.

Step 4: Return to *Step 2* until the main iteration tolerance is reached. Generally the loop voltage profiles could be used to check if formula (4) is satisfied.

Step 5: An additional forward approach using formula (2) is used to calculate the voltage profiles of the radial branches based on the loop and feeder voltage distributions.

Since the Newton-Raphson method has robust convergence ability for strong connected networks, and the main part of the high R/X ratio radial type system is handled by the proposed ratio-flow, which is basically a ladder network method with inherent robust convergence ability for ill-conditioned systems, the overall robustness of the hybrid distri-flow algorithm can be expected.

Test models, results and analysis

Several test systems are used to test the convergence performance of the proposed methods in MATLAB, compared with a forward method and a standard Newton-Raphson method[16] for different radial system configurations (long/short feeders, laterals, complex structures) under different load conditions (uniformly distributed load, heavy ending load, polynomial load).

Convergence characteristic analysis of ratio-flow for different R/X ratio networks under different load conditions in a 13-bus radial system with constant power load model

A 13-bus radial system is shown in Fig. 6, which has a main feeder and two lateral branches. The analysis is performed with a serial power flow analysis with different system parameters and loading conditions:

- let the system load be uniformly distributed and change the feeder impedance R/X ratio from 3 : 1 to 6 : 1 (Table 1);
- let the system load be uniformly distributed and vary the lateral impedances (Table 2);
- increase the loading level at lateral branches and feeder nodes (Table 3).

Tables 1–3 record the numbers of iterations. The results show that, under the above different system conditions, ratio-flow maintains an obvious advantage on the convergence speed over the forward flow method.

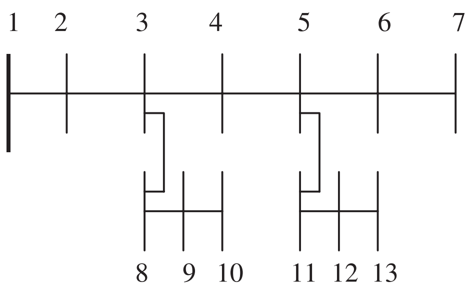

Fig. 6 *A 13-bus test system with 1 feeder and 2 laterals.*

TABLE 1 *Effect of R/X ratio on convergence speed*

Method	$Z = 0.09 + 0.03j$	$Z = 0.12 + 0.03j$	$Z = 0.15 + 0.03j$	$Z = 0.18 + 0.03j$
Ratio	4	3	4	6
Forward	5	4	6	7

TABLE 2 *Effect of lateral impedance on convergence speed*

Method*	Case 1	Case 2	Case 3	Case 4
Ratio	4	4	4	4
Forward	4	5	6	6

*Case 1-Lateral and feeder have the same impedance, $Z = 0.04 + 0.02j$;
Case 2-Increase the impedance of lateral 5-11-12-13 by 50%, $Z = 0.06 + 0.03j$;
Case 3-Increase the impedance of lateral 5-11-12-13 by 100%, $Z = 0.08 + 0.04j$;
Case 4-Increase the impedance of lateral 5-11-12-13 & 3-8-9-10 by 100%, $Z = 0.1 + 0.04j$.

TABLE 3 *Effect of loading levels on convergence speed*

Method*	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Ratio	5	6	5	5	5	6
Forward	7	8	7	7	6	7

*Case 1-Heavy load at one lateral end, node 10, $S10 = 0.26 + 0.04j$ p.u.;
Case 2-Heavy load at lateral ends, node 10 and 13, $S10 = S13 = 0.26 + 0.04j$ p.u.;
Case 3-Uniformly heavy lateral load, $S = 0.16 + 0.04j$ p.u.;
Case 4-Heavy load at feeder sending end, $S1 = 0.36 + 0.08j$ p.u.;
Case 5-No load at lateral connection points (node 3 and 5);
Case 6-Heavy load at feeder end node 7, $S7 = 0.36 + 0.08j$ p.u.

Ratio-flow performance in a practical 30-bus distribution system with polynomial load model

A practical 30-bus distribution system with polynomial load model is used for the performance analysis of ratio-flow (Fig. 7). The system data can be found in Refs.[4,6,14] The polynomial load model expressed as formula (1) is applied for the 30-bus system. Different load models are tested with the proposed ratio-flow algorithm, as shown in Table 4. In Table 4, model 1 is the constant power model, model 6 is the constant current model and model 7 is the constant impedance model.

The following conclusions were obtained from the 30-bus system:

- With a long lateral branch, high R/X ratio and different line impedances on feeder and lateral sections, this system shows serious ill-conditions for the

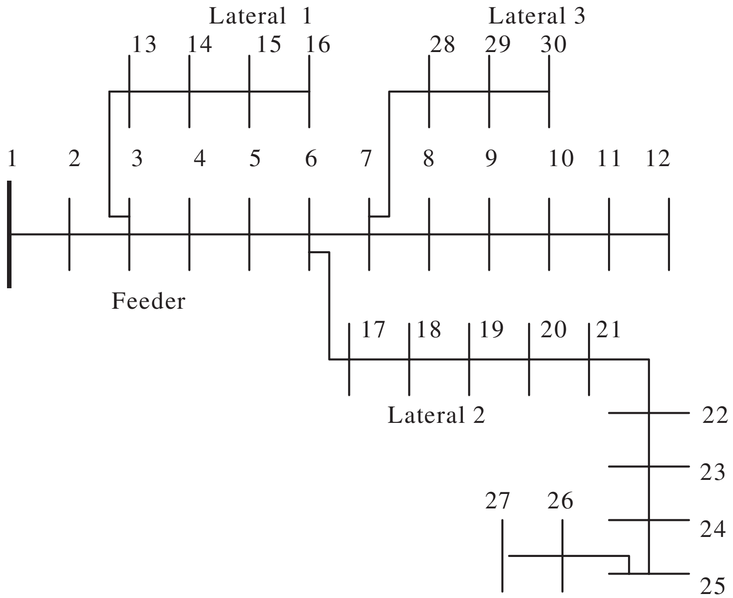

Fig. 7 *A 30-bus distribution system.*

TABLE 4 *Polynomial load model for 30-bus system*

Mode	No. of	a	a	A	a	b	b	b	B
1	Iterations	0	1	2	3	0	1	2	3
1	5	1	0	0	0	1	0	0	0
2	5	0.8	0.2	0	0	0.8	0.2	0	0
3	4	0.6	0.4	0	0	0.6	0.4	0	0
4	4	0.5	0.5	0	0	0.5	0.5	0	0
5	4	0.4	0.6	0	0	0.4	0.6	0	0
6	4	0	1	0	0	0	1	0	0
7	4	0	0	1	0	0	0	1	0
8	3	0.5	0.2	0.2	0.1	0.5	0.2	0.2	0.1

standard Newton-Raphson method. The standard Newton-Raphson program used earlier failed to reach convergence on this system.

- By introducing the polynomial load model, the power flow solution could be more precise compared with the traditional constant load model. The proposed ratio-flow method integrates the polynomial load current adjustment in the whole system iterations and reaches convergence quickly (Table 4). The power flow accuracy is the same as that in Ref.[14]

Fig. 8 shows the feeder voltage profile of the 30-bus system with different load models obtained from ratio-flow. It shows that the accurate load model is an essential factor of accurate power flow solutions.

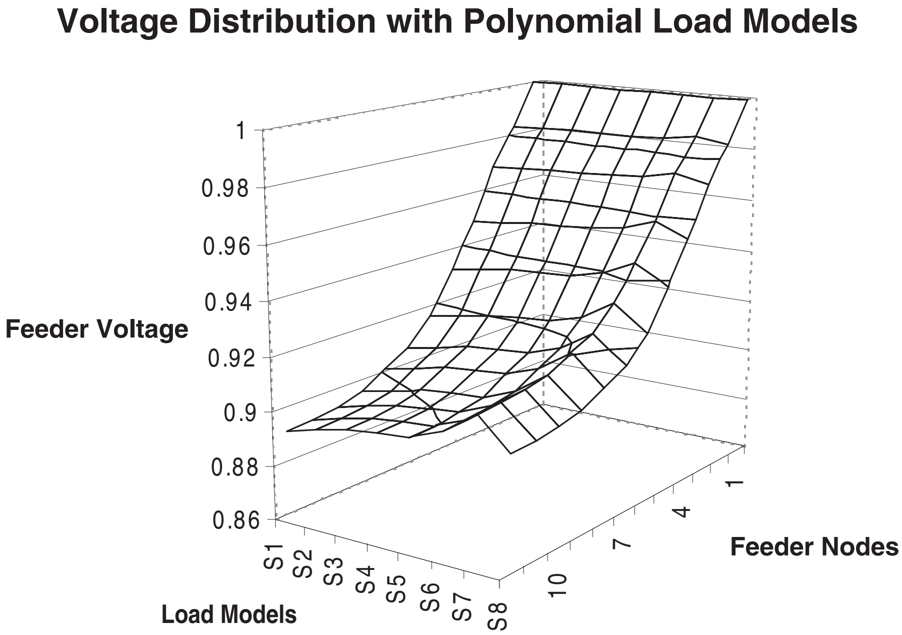

Fig. 8 *Feeder voltage profile with different polynomial load models.*

Sample complex distribution system power flow analysis with the proposed hybrid distri-flow

The advantages of ratio-flow are evident from the above analysis. These improvements can also be applied to a complex distribution system with multiple sources and small loops, as well as radial subsystems. A 15-bus system shown in Fig. 9, which has two sources in a 3-bus loop and a 13-bus radial part, is used as the sample system for the proposed hybrid distri-flow. For the sake of comparison with the Newton-Raphson method, only a constant $P - Q$ load model is applied. The use of the polynomial load model, which is the main strength of the ratio-flow method, is not demonstrated here because the Newton-Raphson method is not suitable for such a case.

The power flow results (bus voltages) of the hybrid distri-flow method and the standard Newton-Raphson method are compared in Table 5. We find that the proposed hybrid distri-flow is suitable for multiple-source distribution systems.

Conclusions

This paper presents a new distribution power flow algorithm which is quite robust, adaptable and efficient:

(1) The ratio-flow algorithm has inherently strong convergence ability and very attractive convergence speed which can be compared to the Newton-Raphson

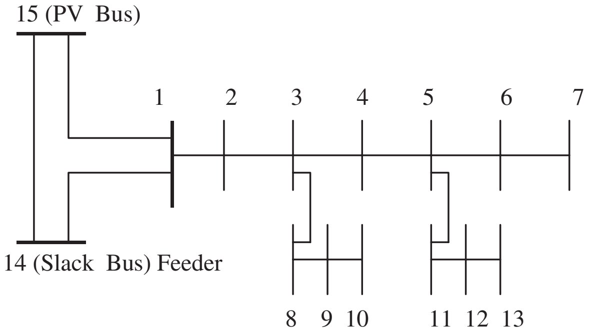

Fig. 9 *15-bus sample complex system.*

TABLE 5 *Power flow result of 15-bus system (with the R/X ratio of the radial part as 3 : 1)*

	Newton-Raphson method		Hybrid distri-flow	
Bus no.	Voltage magnitude	Angle degree	Voltage magnitude	Angle degree
1	1.000	–0.224	1.0003	–0.2235
2	0.992	–0.836	0.9916	–0.8369
3	0.984	–1.406	0.9837	–1.4077
4	0.979	–1.781	0.9786	–1.7832
5	0.974	–2.105	0.9742	–2.1053
6	0.973	–2.226	0.9727	–2.2262
7	0.972	–2.293	0.9718	–2.2929
8	0.982	–1.554	0.9813	–1.5539
9	0.98	–1.649	0.9799	–1.6489
10	0.98	–1.703	0.9792	–1.7031
11	0.972	–2.257	0.9718	–2.2571
12	0.971	–2.329	0.9707	–2.3291
13	0.971	–2.348	0.9703	–2.3484
14	1	0	1	0
15	1.005	–0.144	1.005	–0.1439

method under most load and network conditions. Moreover it is also quite suitable for the polynomial load model which Newton-Raphson like algorithms are usually not suitable for.

(2) The hybrid distri-flow algorithm combines the advantages of the Newton-Raphson method and the proposed ratio-flow so that the complex distribution power flow problem could be easily solved.

(3) The proposed methods are inherently suitable for distributed computing and real-time power flow solutions. The computation cost and the data transfer between computation modules are the least in the present power flow methods.

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