
Teaching electrical engineering using Maple

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Abstract Many electrical engineering (EE) students have difficulty in learning technical subjects because they lack sufficient competence in mathematical modeling and in algebra. Maple is a powerful program for doing symbolic algebra, numerical calculation, and plotting of graphs, so using this program allows students to spend more time on modeling and interpreting results. Maple also has a text editor, which makes it feasible to require students to explain their results in writing. The design of Maple documents suitable for EE teaching is discussed; a standard format, including bibliographical information, is recommended for easier use.

Keywords CAS; education; Maple; WAC

This author's experience and current literature show a widespread problem in electrical engineering (EE) education, namely that many students do not master the mathematical tools that are prerequisite for studying EE. This lack of mathematical knowledge makes it difficult for them to analyze signals and circuits, a task that is heavily dependent on mathematical modeling and manipulation. Problem solving from first principles in electrical engineering often starts with a verbal statement of the physical problem, and the solution can be regarded as a four-step process. The student's first task is to state the problem in mathematical terms, i.e. to formulate the equations representing the physical situation. The second step is to symbolically solve those equations for the desired terms. The third step is the easy part: plugging in the values and doing the numerical calculations. Fourth, and frequently overlooked by many students, is to judge the correctness of the initial equations, the analytical solutions, and the final numerical results. In short, does the answer make sense?

Students frequently have difficulties stating a physical problem in mathematical terms. In addition, they often lack the ability to do the symbolic manipulations necessary for solving the relevant equations. As a result, they tend to avoid solving problems from first principles, preferring rather to choose a likely-looking (but often the wrong) formula from a textbook, put in numerical values, and get some numbers. Although unintentionally, I'm sure, textbooks and lectures aggravate this problem by placing great emphasis on deriving general formulas for use in numerical calculations. And students learn the wrong lesson: that the important thing is to memorize formulas and plug in numbers.

The third step of problem solving, numerical calculation, has for decades been done by calculators and computers. The second step, symbolic manipulation, often turns out to be the main focusing point in engineering subjects. EE classes have a tendency to become remedial classes in algebra. Although students must have the means of doing the second and third steps in problem solving, as all steps are necessary to obtain and verify a solution, this is not the essential part of studying EE.

To really learn EE, students must learn to do symbolic analysis of circuits, the first and the last steps in problem solving, and this should be the main focus of our teaching effort.

One problem we have as teachers is therefore how to handle the situation that many students have a limited ability to do symbolic manipulations. The literature is full of good suggestions, one of which is to have students use Computer Algebra Systems (CAS). CAS, such as Derive, Mathematica and Maple have been used in college education for several decades¹ with good pedagogical results.² Early use of CAS in teaching focused on mathematics education, but in recent years CAS have also been used in teaching other subjects as well, including physics,³ chemistry⁴ and electrical engineering.^{5,6} Computers have, of course, been ubiquitous in EE education since the beginning of the computer age, but their use has focused on numerical calculations, simulations and computer-aided design. Numerically based tools, such as SPICE, are useful in many situations; but, where students are concerned, they often obscure an important fact – that behind any numerical result is a symbolic model of the physical world.

Maple V, as one of several advanced CAS systems, is well suited for mathematical analysis of electronic signals and circuits. In addition to doing symbolic algebra, it can plot graphs and do numerical calculations. Thus Maple promises to remove a lot of the drudgery involved in the second and third steps of problem solving, and leave more time to focus on the first and the last steps. Maple combines these mathematical capabilities with a nice text editing function that substantially enhances the usefulness of Maple as a teaching tool. Text can be included and can, together with calculations, be saved in an electronic document usually referred to as a worksheet. The possibility for teachers and students to write extensive worksheets including both advanced mathematical modeling and textual explanations should be taken advantage of. This means employing another popular educational technique that will benefit the students, namely writing as a way of learning.⁷

A model worksheet and its use

At Gjøvik College, Maple V has been used for several years in teaching mathematics, physics and EE, and as a result many Maple worksheets have been written. We use essentially three different kinds of worksheets. Supplementary lecture notes, like the one presented in this paper, have been distributed to the students. Students have been required to hand in a number of homework assignments in the form of Maple worksheets. Solutions to homework problems completed by the teacher, or by a student, have been distributed by posting them on the web page of the particular course. Worksheets written by people at other institutions have also been used.

The result of this activity is a large number of worksheets. Experience gained in this work has shown the importance of paying attention to details in Maple worksheet design. To sum up our experience, a worksheet is presented in the Appendix. It forms a model for similar worksheets that are distributed to our students or that the students are expected to hand in as part of their homework.

Writing good worksheets is a time-consuming endeavor, and use of Maple would become even more appealing if such documents could be shared among teachers and students at different institutions. Distribution of Maple documents among institutions is another reason to pay careful attention to the details in worksheet design.

The layout of a worksheet will vary according to taste. However, a sensible division of the worksheet into different sections is a must. What follows is the presentation of a worksheet written according to a standard that I have found useful in my own teaching and that I have encouraged the students to use. The complete worksheet is listed in the Appendix.

Bibliographical information

Any worksheet that will be used by more than one person must have a section containing bibliographic information. Experience and reason tell us that students as well as teachers soon forget oral information about a worksheet, so such information needs to be available in writing, on the document.

Each document must have a descriptive title so that the user can refer to it easily. If it contains the solution of a problem given in a textbook, it may be a good idea to make the title of the worksheet some combination of the author's name, the title of the book, and the problem number. Finding descriptive titles is more of a problem than it may first appear, especially in a set of connected worksheets. Often it is useful to have a subtitle that identifies the worksheet as one in a series of several related worksheets. I have had to change titles on a set of worksheets more than once in order to get a coherent system. The reason for including the name of the author of the worksheet below the title should be obvious.

Teachers as well as students may have questions or comments about the contents of a worksheet and may wish to contact the author. Postal or, more appropriately, an E-mail address is therefore necessary information in a worksheet. I once distributed a borrowed worksheet that did not contain an address; the next day a student asked for the office of the author who was a continent away.

The date and version number of a worksheet are also important information. If the worksheet is to be used in teaching, it will sooner or later (usually sooner) need to be modified, and it will be very easy to lose track of which is the latest version of the document.

Another problem is that teachers and students often lose track of their Maple release number. At our school we use Maple V Releases 2, 4, 5 and 6. All releases remain available because worksheets were written for the different releases and it is tedious to upgrade all these worksheets. As other CAS programs (Mathematica) may also be used, worksheets should contain both the name of the program and the release number.

It is easier to find the right worksheet by leafing through a set of paper copies than to find it among computer files. The name of the file where the worksheet is stored must therefore be included in the document. As soon as I get more than 10 worksheets, I lose track of the contents and the corresponding file no matter how carefully I choose the file name. The choice of proper filenames requires some

thought, and to make informative filenames out of a limited number of characters becomes difficult when there are large number of files. *Very_Long_File_Names_Is_Not_Very_Convenient*. The obvious possibilities are to use a name related to the subject of the worksheet or to the course where it is used. I prefer the former because a worksheet quite often is used in more than one course, and then a reference to a particular course is confusing.

In some circumstances it may be necessary to use one or more external library functions in a worksheet. I regularly use a plotting package that is placed in a private library. In order for users, especially outside one's own institution, to use such worksheets it is important to inform them that the document uses one or more routines in a particular library package and where this package is available on the Internet. In my sample worksheet (see Appendix) this is done with a reference to the external library *EEplot4* at the web address: <http://www.hig.no/avdeling/ea/maple/lib/EEplot/>.

For pedagogical reasons worksheets designed for education should encourage students to make modifications. One may assign additional problems for students to do, or there may be notes they themselves want to add to clarify a point. Soon there will be many different versions of the same original worksheet, and it becomes important to know who made any particular modification. I encourage my students to modify the worksheets and strongly urge them to write their name in the «modified by» space. In this way the students get a more personal attachment to the document.

The above information should be kept in a single region at the beginning of the worksheet so that it can be collapsed and thus largely hidden while the students work on the main part of the document. However, it is important that the region be expanded when a printed copy of the worksheet is made so that the information becomes visible.

Pedagogical considerations

Every worksheet should have an introduction. The content of this introductory section may differ according to the purpose of the worksheet. For supplementary class notes, some theoretical presentation of the subject at hand is called for. Use adequate references to keep this introduction to a short review of the theory. If the worksheet is an answer key for a homework assignment, then the problem to be solved should be stated in the introduction. To refer only to a problem number in a textbook is inadequate. Also, the homework that students hand in should have the problem accurately stated in the introductory section. To save students the time of copying the problem, a worksheet containing only the problem could be made available electronically.

The main part of the worksheet could be divided into two sections. The first section would be for the theoretical analysis, which should be kept symbolic if at all possible. The second section is then reserved for numerical calculations, presenting and discussing the results, and checking the correctness of the answers.

Any symbolic calculation should always start from first principles. In EE that often means Ohm's and Kirchhoff's laws. This is very important – students never

get enough experience in applying these laws to different physical situations. The additional work that would result if these calculations were to be done by hand is largely eliminated by the use of Maple. Although it is tempting to start with what teachers or students believe to be a well-known formula for a given class of circuits, it should not be done. Well-known formulas are useful as intermediate checks of the results, but not as a starting point for analysis in a pedagogical situation. After all, much of education is repetition.

Pedagogical considerations also make it important to keep calculations symbolic as long as possible. For instance, symbolic expressions show the relationship between components in a circuit, whereas a numerical value gives no indication of any such dependencies. Symbolic results of a calculation, for example a transfer function, are therefore important for students to better judge and evaluate the validity of the obtained results. Students have a tendency to do numerical calculations using calculators and to resist doing symbolic manipulations. If numerical answers to a problem are required, as is often the case, they should be obtained and checked at the end. A worksheet written by a teacher should therefore stress the importance of doing the symbolic analysis before obtaining the numeric results, and as teachers we should always require the same of our students.

Another important point: students should always be required to judge the correctness of the obtained results. It can be done in different ways depending on the particular subject, but limit estimates are very suitable for many circuits where transfer functions are involved; and Maple will happily do limits. Such checks have a dual purpose in education. First, the symbolic solution along with the estimates provides a deeper insight into the behavior of a circuit and is therefore an important learning experience. Second, checking results is important in any workplace, and it should be practised with regularity.

Worksheets need to encourage students to do further work on a subject. Thus a worksheet might contain a series of questions and problems for students to solve. I have used two ways of posing questions in a worksheet. First, the worksheet should have short questions to help students focus on important aspects of the problem; this should result in students modifying and adding to the document. Questions could require students to elaborate further in writing on a point of understanding, or could require them to do calculations on similar problems or other aspects of the same problem. I have found that such questions require a standard form that students will recognize immediately from one worksheet to the next, so I have used a double question mark and a bold italic type.

Second, a set of elaborate problems relating to the topic could be posted at the end of the worksheet. These should be designed so that students are required to work on a problem in several ways, theoretical, numerical and practical. In order to encourage this I have in the example included problems that further focus on the topic of impedance matching. These problems require the student to do further work in Maple, to do simulations on a numerical tool like PSpice, and finally to test the actual circuit in the lab.

In EE it is important to represent circuits by drawings or schematics (Fig. 1 in Appendix). Maple is not a graphics package, and suitable drawings cannot be made

in Maple. However, schematics can be generated in a circuit program like *MicroSim Schematics*, and then cut and pasted into Maple. *MicroSim* (Fig. 2 in Appendix) is usually more convenient than regular drawing programs, such as *Paintbrush*. It gives a circuit representation that is, or should be, familiar to students, and it is also much easier to use.

Good instructional material contains references to other literature so that the reader can easily find additional sources to shed light on the subject from other points of view. It is important that a worksheet too contains such a list of references to relevant literature. Although the courses in which our worksheets are used include textbooks, it is necessary to encourage students to use the library in order to get different perspectives and hopefully better understand the topic.

Finally, it is not realistic to expect students to learn more than a limited number of Maple commands, and even these must be repeated frequently. Thus it is important that the worksheets to be used by students contain, as far as possible, only this limited number of commands. The specific commands will vary from subject to subject, but teachers should consider this problem before they use Maple in teaching. Unusual commands used in, or output from, a worksheet may be explained in the text for the benefit of the students. In the Appendix the expression *RoorOf* is briefly explained.

Writing Across Curriculum (WAC)

A pedagogical method that has recently gone through a revival and become popular is to have students write about the subject they are studying, for the sake of learning.⁸ This method is often referred to as WAC (writing across curriculum) and has been combined with Kolb's learning-style theory.⁹ If students have to formulate in words what they are working on, it helps them build cognitive structures in their minds. Obviously students cannot write sensibly about a subject if they don't know it reasonably well. Writing can therefore be an important way of learning a subject, and this possibility should be exploited. Maple, with its reasonably good text editor, can combine mathematical work in engineering and WAC into one document, and students as well as teachers should use this for learning and teaching purposes.

When solving problems using Maple, students should be required to write down in detail their reasons for formulating the equations. They should also have to explain how to solve these equations if they had to do it by hand. When checking the mathematical results obtained by Maple, students should be required to explain in writing why they believe the results must be correct. In fact, anything that is relevant to solving a problem and interpreting the results should be written down.

Maple is an ideal tool for doing these tasks, and the problems set out in the sample worksheet are formulated so that students are required to do a fair amount of writing in addition to the mathematical work. Homework is for students to learn from, and explaining things in writing forces them to think through the subject matter. Writing their own worksheets as an alternative or a supplement to taking class notes is also valuable. Writing, apart from being a learning tool, is also a necessary skill for any engineer in the workplace, and should be practised at any opportunity. Any assign-

ment given should be considered a small project; it therefore requires a small report written according to accepted standards.

The World Wide Web has given students and teachers a whole new environment to work in. It is common that each course has its own Web page, and using Maple becomes easier for students if worksheets stored in these pages are set up with the correct version of Maple as browser. Assignments may be formulated in a Maple worksheet and made available for students to work on and complete for later submission. Posting a student's worksheet as an answer key to homework problems is encouraging for those students who get their documents posted.

All this work sounds like a lot to expect from a worksheet, especially one written by a student. However, it is better, I believe, for a student to do a few problems in great detail than many problems more superficially. And experience tells me that a majority of the students are willing to put more effort into making extensive worksheets than into regular handwritten reports. It looks more professional and is more easily modified.

Summary and conclusion

Use of CAS like Maple adds a new and powerful tool to EE education. It has the ability to reduce the drudgery of symbolic manipulations that students find so tedious. Maple also includes graphic and numeric capability which, although it may be inferior to more specialized packages, is with some exceptions more than adequate for most students. Maple also contains a decent text editor that makes it possible to write mathematical expressions, figures and text in a single document. However, if Maple worksheets are to be used successfully in education it is important that simple design guidelines be observed to make them easier for students and teachers to use. Worksheets must have comprehensive bibliographic information such as a descriptive title, name of the author, address and E-mail address of the author, date, version number, file name of the worksheet, and Maple release number. If a worksheet makes use of procedures in special libraries or files, the place on the Internet where these files or libraries can be found needs to be included.

For pedagogical reasons, calculations in worksheets should start from first principles, and be kept symbolic as far as possible. Strong emphasis should be placed on assigning students problems that enlarge the worksheet. A key to effective learning, I believe, is that students should concentrate on modeling and testing results. To do this they should include both mathematics and written discussion in the worksheet. Having students write as a way to learn is strongly emphasized, and Maple is very well suited for this purpose.

All in all I find Maple a most exciting tool to use in EE teaching, and I am sure that all the possibilities have not yet been exploited. Maple or other similar CAS could become a unifying program in EE education, replacing a string of other programs from word processors to numerical simulation tools. This replacement should be seriously considered, since one of the main complaints from students is that there are far too many different programs to be learned, sometimes two or three in one course, and that these programs are used in one course only.

References

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Appendix

IMPEDANCE MATCHING NETWORK

Supplementary Notes in Electronic Circuits #3.1

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Information

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Date: June -97 Version: 1

File: IMPMACH1.MWS Maple V Release 4

Extern. Lib.: EEplot4 at <http://www.hig.no/avdeling/ea/maple/lib/EEplot/>

Modified by: Nobody

Introduction

This worksheet contains a brief introduction to the theory of a simple impedance matching network, along with some theoretical and practical student problems.

Assume a given signal source with a given impedance R_2 and a load R_1 , both impedances being real. In order for the maximum power to be transferred from the source to the load, R_1 must be equal to R_2 . If the two impedances are different we may insert an impedance matching LC-network between the source and the load (Krauss *et al.*, 1980; Franke, 1991). We will construct an impedance matching network starting with a circuit as outlined in Fig. 1.

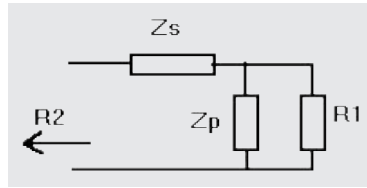


Fig. 1

We make no assumptions about Z_s and Z_p except that they are imaginary, so $Z_s = jX_s$ and $Z_p = jX_p$. For matching to occur, the impedance of the network as seen from the terminals must equal the source impedance $R2$. This will insure that the maximum energy will be transferred from the source to the load resistance $R1$, since Z_s and Z_p have purely imaginary impedances and thus can absorb no energy.

□ Derivations and Calculations

□ Derivations

The impedance of this network between the terminals is equal to the impedance of the source (Franke 91, Krauss *et al.* 80); thus we have the following equations:

> restart;

> E1:=R2=Zs+Zp*R1/(Zp+R1); Zs:=I*Xs; Zp:=I*Xp;

$$E1 := R2 = Zs + \frac{Zp R1}{Zp + R1}$$

$$Zs := I Xs$$

$$Zp := I Xp$$

> E2:=evalc(E1);

$$E2 := R2 = \frac{Xp^2 R1}{R1^2 + Xp^2} + I \left(Xs + \frac{Xp R1^2}{R1^2 + Xp^2} \right)$$

We have obtained an equation relating the unknowns X_s and X_p to the known resistances $R1$ and $R2$. Since this is a complex equation, the real and the imaginary parts must be equal, so we may rewrite equation E2 as follows.

> Eq1:=evalc(Im(lhs(E2)))=evalc(Im(rhs(E2)));

$$Eq1 := 0 = Xs + \frac{Xp R1^2}{R1^2 + Xp^2}$$

> Eq2:=evalc(Re(lhs(E2)))=evalc(Re(rhs(E2)));

$$Eq2 := R2 = \frac{Xp^2 R1}{R1^2 + Xp^2}$$

We have obtained two purely real equations with unknowns X_p and X_s , and we shall solve these equations with respect to the two unknowns. Solving Eq1 and Eq2 gives.

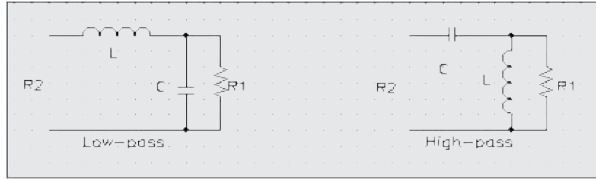


Fig. 2

> solve({Eq1,Eq2},{Xs,Xp});

$$\{Xs = \text{RootOf}((R2 - RI)Z^2 + R2, \text{label} = _LI)(R2 - RI), Xp = \text{RootOf}((R2 - RI)Z^2 + R2, \text{label} = _LI)RI\}$$

> assign("");

((RootOf indicates that there is more than one solution to the equations. The individual solutions may be found by giving the command 'allvalues'.))

> Xs:=[allvalues(Xs)]; Xp:=[allvalues(Xp)];

$$Xs := [-\sqrt{(-R2 + RI)R2}, \sqrt{(-R2 + RI)R2}]$$

$$Xp := \left[\frac{\sqrt{(-R2 + RI)R2RI}}{-R2 + RI}, -\frac{\sqrt{(-R2 + RI)R2RI}}{-R2 + RI} \right]$$

From these expressions we see that $R1 > R2$, otherwise Xp and Xs would be imaginary – contrary to our assumptions. Furthermore we have two sets of solutions, where a positive Xs gives a negative Xp and vice versa. Note that it is the first root of Xp that is negative, since $(R2 - R1) < 0$.

We get two different circuits that fit our requirement, each with a coil (L) and a capacitor (C) as shown in Fig. 2. Remember that positive X means a coil and negative X means a capacitor.

These circuits provide impedance matching between the source and the load, but at only one frequency. The reason for this is that L and C are frequency dependent. As function of frequencies, these two circuits are filters, one LP and one HP.

?? Make sure you understand why these circuits are LP and HP filters, respectively, by performing limit estimations.

We proceed to find expressions for L and C for the HP circuit.

> E1:=1/(wo*Chp)=-1*sqrt(R2*R1-R2^2);

$$E1 := -\frac{1}{wo Chp} = -\sqrt{R2RI - R2^2}$$

> Chp:=solve(E1,Chp);

$$Chp := \frac{1}{\sqrt{R2RI - R2^2} wo}$$

```
> E2:=wo*Lhp=-R1*sqrt(R2*R1-R2^2)/(R2-R1);
```

$$E2 := woLhp = -\frac{R1\sqrt{R2R1 - R2^2}}{R2 - R1}$$

```
> Lhp:=solve(E2,Lhp);
```

$$Lhp := -\frac{R1\sqrt{R2R1 - R2^2}}{wo(R2 - R1)}$$

?? Note that L and C are functions of R1, R2 and the frequency wo. Make sure you understand why this is so. Simplify these expressions somewhat by hand. Find similar expressions for the LP circuit.

□ Calculations

Consider the HP circuit in Fig. 2, and choose values for the components so the impedance of the circuit as a function of frequency may be plotted.

```
> Zp:=1/(1/(s*L)+1/R1); Zi:=1/(s*C)+Zp;
```

$$Zp := \frac{1}{\frac{1}{sL} + \frac{1}{R1}}$$

$$Zi := \frac{1}{sC} + \frac{1}{\frac{1}{sL} + \frac{1}{R1}}$$

Use the following values:

```
> R1:=200; R2:=150; wo:=evalf(2*10^6,3);
```

$$R1 := 200$$

$$R2 := 150$$

$$wo := .200 \cdot 10^7$$

Numerical values for the capacitor (C) and the coil (L), are therefore:

```
> C:=evalf(Chp,3); L:=evalf(Lhp,3);
```

$$C := -576 \cdot 10^{-8}$$

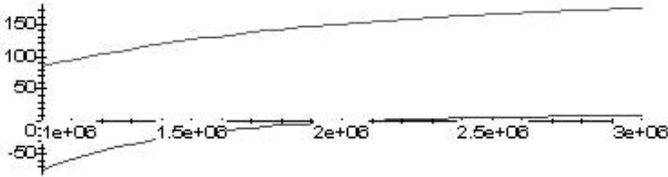
$$L := .000173$$

?? Given these values for C and L, analyze the circuit as seen from the load resistance R1. Show that this impedance is equal to R1 so that the circuit also acts as an impedance matching network from R2 to R1.

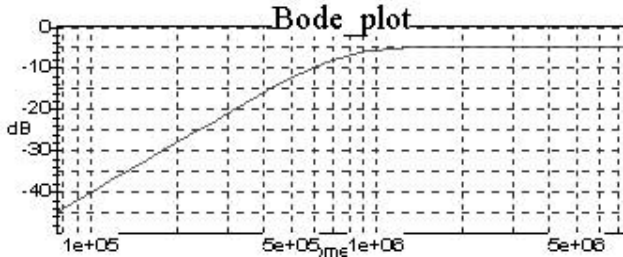
Plot the real and imaginary parts of the impedance as a function of frequency. If done correctly, we expect to get a purely real impedance (R2) at the designed frequency. We expect the imaginary part of the impedance to be equal to zero.

```
> s:=I*w;
```

$$s := Iw$$



Plot 1



Plot 2

```
> Zre:=evalc(Re(Zi)); Zim:=evalc(Im(Zi));
```

$$Zre := \frac{1}{200} \frac{1}{\frac{1}{40000} + \frac{3341240937 \cdot 10^8}{w^2}}$$

$$Zim := -.1736111111110^9 \frac{1}{w} + \frac{5780346521}{w \left(\frac{1}{40000} + \frac{3341240937 \cdot 10^8}{w^2} \right)}$$

```
> plot({Zre,Zim}, w=10^6..3*10^6);
```

Plot 1 shows that the real and imaginary parts of the impedance behave as expected.

?? Explain the graphs in Plot 1. Why does the imaginary part of the impedance change sign at the design frequency?

Finally let us find the Bode plot for the HP circuit (Fig. 2) in order to investigate the behavior of this filter. Plot 2 confirms that the circuit is a HP filter.

```
> s:='s';
```

$$s := s$$

```
> Z1:=R2+1/(s*C); Z2:=R1*s*L/(R1+s*L);
```

$$Z1 := R2 + \frac{1}{sC}$$

$$Z2 := \frac{R1sL}{R1 + sL}$$

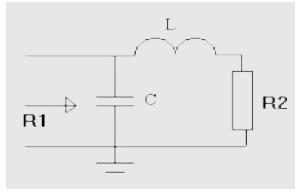


Fig. 3

```
> H:=simplify(Z2/(Z1+Z2));
```

$$H := 346000 \frac{s^2}{6003472222 \cdot 10^{12} s + 605500 s^2 + 3472222222 \cdot 10^{18}}$$

```
> with(EEplot4):
```

```
> Bodeplot(H);
```

□ Problems

* Find the transfer function for the LP circuit (Fig. 2) with the output measured across R1. Plot the amplitude and phase of this function when $R_2 = 75 \text{ Ohm}$, $R_1 = 150 \text{ Ohm}$, and $f_0 = 500 \text{ kHz}$.

* Plot the impedance (real and imaginary or amplitude and phase) of the LP circuit as a function of frequency. Compare this to the impedance plots for the HP circuit already found.

* Do a similar analysis for the circuit in Fig. 3.

* Assume that one of the resistances R1 or R2 equals 75 Ohm, and the other 150 Ohm. Determine R1 and R2. The circuit should operate at 10 kHz. Determine L and C.

* Find the transfer function for the circuit as seen from R1 and plot the frequency response.

* Simulate the circuit using Spice and determine the frequency response and the input impedance of the circuit at the design frequency.

* Construct the circuit and measure the frequency response and impedance of the circuit at the design frequency. Measure the circuit seen from R2 when the input is terminated by R1.

* Compare the results obtained by the three different methods.

* Use the circuit in Fig. 3 but exchange the values of R1 and R2. What will the results be?

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