
The mathematics background of undergraduate engineers

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Abstract A major problem for engineering undergraduates is a lack of basic skills in number and algebra. While we seek middle-term solutions to these problems we must do something for the immediate term. It is suggested that the other engineering subjects be taught in the first semester in a qualitative way.

Keywords engineers; mathematics; undergraduates

The teaching of mathematics to engineers was a matter for debate as long ago as the turn of the last century. This debate has continued undiminished over the last one hundred years. The issue is perceived by many engineers and mathematicians to be a long-running source of tension. Part of the problem has been that, for many years, some of those mathematics departments which had the responsibility for teaching the mathematics component of the engineering degree tended to provide a watered-down version of modules (in the parlance of today) which were being taught to their own students. Mathematics, it was argued, was a subject in its own right and any applications should be left to the engineering lecturers; a view, it has to be said, that was shared by some engineering staff. This was in sharp contrast to the opinion of the students, whose main complaint was the lack of any attempt to demonstrate the applicability of those areas of mathematics which they were being taught. Most mathematics departments today do not take this approach; however, even where there is such enlightenment, the teaching of mathematics is being taken into the engineering departments, often under some semi-transparent title such as 'quantitative methods'. Sometimes there are financial reasons behind the reclaiming of the mathematics modules and sometimes there is the wish to have control of the syllabus in the belief that this will correct the problem of students not seeing the relevance of the mathematics being taught.

Another part of the problem is that mathematics has been seen as a necessary evil by some engineering staff as well as by their students. Such attitudes militate against a close liaison between mathematics staff and engineering staff in order to present mathematics as one of the major engineering subjects, if not the core subject. To be successful, mathematics must be taught as an engineering subject, and not as a peripheral. Certainly, in the first two years mathematics should not be dispersed among the other engineering subjects on an 'as and when' basis.

All this would be sufficient cause for concern, but there is the added dimension of what appears to be a steady decline in the mathematical knowledge and skills of successive cohorts entering undergraduate degree programmes in engineering. There is dissatisfaction in many engineering departments at the mathematical preparedness

of entrants to engineering degree courses. Two symptoms in particular are manifest: the diversity of the intake and the lack of basic skills in number and algebra. These symptoms are discussed in the next two sections.

Diversity of the intake

In a first year class of engineering students it is not uncommon to find some with good grades in 'A'-level mathematics alongside some with poor grades and some with various vocational qualifications, typically BTEC at Level III and Advanced GNVQ. Often there are some students whose most advanced mathematics qualification was a poor grade, achieved two years earlier. In the worst cases some of these students cannot add, subtract, multiply or divide numerical fractions, cannot recognise or plot simple straight-line graphs, and have no skills in algebraic manipulation. It is frequently the case that students coming from a BTEC or Advanced GNVQ background have had little, if any, exposure to differential and integral calculus – basic tools which are required from the very start of an engineering degree programme.

There have been well-meaning attempts to remedy some of these problems. These include reducing syllabus content, which, although helping the weaker students, undoubtedly disadvantages the better ones and makes these less prepared for the mathematical demands of more advanced and analytical engineering courses. Some universities have established support centres where students experiencing difficulties can pay a visit for supplementary resources and additional tuition. However, those working in such centres know that this is not getting to the root of the problem. Students who enter university cannot, by visiting a support centre for just a few weeks, at the same time as their engineering courses are proceeding, make up for the fact that they have not even heard of the terms derivative, or integral. Students struggling with the algebra of complex numbers are often hindered by their lack of basic algebraic skills and it is very difficult to try to teach these skills concomitantly.

In September 1997 the Engineering Council produced a revised version of its 1990 document on SARTOR (standards and routes to registration)¹ in which it seeks to ensure that engineering qualifications in the UK will continue to compare favourably with the highest standards internationally.

The Engineering Council recognises the importance of mathematics in engineering education. In its summary to SARTOR 3 it records one of its reasons for change as being the reduced 'A'-level coverage of mathematics in relation to the needs of engineering. The author chaired a small working group tasked with preparing a response to the implications for mathematics of the recommendations in SARTOR 3. Nineteen of the engineering institutions responded to an invitation to comment on the proposals and their comments were incorporated in the main in the final report 'Engineering mathematics matters'² published by the Institute of Mathematics and its Applications. The group addressed the need for a different mathematical education to be provided for those aiming for Incorporated Engineer status in contrast to those aiming for Chartered Engineer status. Whilst the proposals will alleviate some

of the problems experienced by today's students, there is a much more deep-rooted malaise which must be faced.

The IMA working group believed that any proposed common core in mathematics for IEng students should be radical in a number of ways. Key features of it are:

- a significant reduction in syllabus content from the traditional course;
- a low-level starting point, allowing a thorough consolidation of basic techniques;
- an emphasis on developing confidence in the application of basic techniques;
- a thorough integration of modern mathematical technologies as tools;
- a motivation through transparent and modern applications;
- a relevance to the career aspirations of an Incorporated Engineer;
- a high-threshold criterion-referenced achievement stated in terms of learning outcomes.

Upon completion of an IEng accredited degree programme a student should:

- be confident in the application of a range of arithmetic and algebraic techniques;
- be competent in the application of a range of arithmetic and algebraic techniques;
- be able to solve problems using simple calculus-based techniques;
- be able to make appropriate use of modern technology, for example computer algebra;
- be able to make appropriate use of spreadsheets;
- have a working knowledge of simple probability and of basic statistical techniques.

There is a danger that, for IEng students in particular, mathematics may be seen as an abstract set of techniques, which bears little relevance to their other studies. Those who teach engineering modules may sometimes reinforce this idea. Consequently the mathematics which is taught should be shown to have applications in fields of engineering which these students are likely to enter.

Practising engineers have access to a full range of modern mathematical technologies when problem solving. The students should therefore be exposed to and be proficient in using technologies such as graphical calculators, computer algebra packages, and spreadsheets. This is not to say that a firm foundation in algebraic and numerical skills is not essential. On the contrary, such a foundation is essential to be an intelligent user of modern mathematical technology. The technology needs to be integrated into the curriculum in such a way that it reinforces rather than replaces the need for this firm foundation.

The proposals contain a much more flexible provision where the mathematical *and* engineering demands made of students are tailored to their level at entry.

The task of the mathematics lecturer has always been a difficult one. Some engineering colleagues would argue that they themselves had coped without 'all this mathematics' whilst others wanted relatively advanced material taught early on. Some wanted complex numbers taught at the outset, some wanted vector methods to be the first topic and yet others demanded that calculus be covered before anything else. Many thought that they could cover the material in less time; most believed that they could do the job better.

There are those who advocate the abolition of a separate set of modules in mathematics. Each lecturer should teach the mathematics which he needs, as and when it is needed. The lesson from Advanced GNVQ is that mathematics cannot be learned successfully via a piecemeal, 'just in time' approach. The fact that a student has a tick in the box 'can solve a quadratic equation' is no guarantee that the task can be repeated successfully when required. Like other engineering subjects, mathematics must be built up steadily and systematically, and it takes time. Unfortunately for the students, it also takes sustained effort on their part.

It is paradoxical therefore that over the last few years the amount of time available to teach the basic mathematics has been, if anything, reduced. Faced with a more heterogeneous input which has a lower mean the process time has been cut. Small wonder then that the output does not live up to expectations.

Are your standards slipping?

It should scarcely be necessary to remark that today's students are not the students of yesterday, although it seems to the mathematics lecturer that his engineering colleagues often forget this, at least as regards the mathematics aspect. In the last decade or so, two factors have conspired to make the task of those who lecture mathematics to engineering undergraduates more difficult.

The main problem facing many students are: a lack of basic skills in number and algebra, engineering modules which assume knowledge and skills which some students do not have and an inability to handle multi-stage problems. The first of these impedes the understanding of those topics which assume familiarity with these areas; the skills and knowledge referred to in the second cannot be acquired easily and quickly.

On the one hand, student numbers have increased dramatically as a direct result of Government directives, yet the number applying to read engineering has decreased. As a consequence, many engineering degree programmes admit students with a level of achievement which would not have been deemed acceptable in earlier years. On the other hand, the level of mathematical achievement of those with apparently comparable 'A'-level qualifications appears to have declined, despite the requirements on entry grade levels being maintained. There are many published articles which support this contention, for example Lawson.³ Lawson showed how the performance on a standard entry test of basic (yes, basic) mathematics revealed an alarming decline in knowledge and skills: the student of 1997 with Grade C in mathematics performed as poorly as the student of 1990 who had achieved Grade N in mathematics. For students who entered via the BTEC or Advanced GNVQ routes the situation was even more desperate.

This state of affairs seems bad enough, but worse is to follow. From September 2000, those who embarked on their post-16 education were able to study a broader range of subjects. There is concern that this will lead to a reduction in the time devoted to each of the (typically) four subjects which are being studied in Year 12. Also, there is pressure on schools to take three of the six 'A'-level modules in Year 12, whereas the practice in many schools had been to take two modules in Year 12

(and four in Year 13), allowing a consolidation of material supposedly learned for GCSE. This could mean that, even by today's standards, future entrants to engineering degree programmes will be less well prepared mathematically.

Just how bad is the situation? Mathematics lecturers, sometimes rightly so, have been accused of teaching at a far too theoretical level, but we are talking here about really basic mathematics. In the work referred to earlier Lawson³ showed that in 1997, 54% of first-year students with Grade C in mathematics could not identify correctly the graph of the cosine function from a selection of four options; amongst students with Advanced GNVQ the figure was a depressing 13%.⁴ This alarming ignorance is apparent across the range of questions in the test.

The results published last August for 'A'-level and 'AS'-level examinations are depressing. There was a small percentage decrease in the number of candidates offering 'A'-level mathematics; of these roughly the same number of candidates passed as was the case in the previous year, but there were fewer candidates obtaining a grade C or above. There were some 9000 fewer candidates offering 'AS' mathematics than offered 'A'-level mathematics (some 57,000 as opposed to 66,000) and of these about 30% failed. This hardly bodes well for next summer.

The results for GCSE mathematics have come under criticism after a senior examiner alleged that grade C was being awarded to candidates with as little as 18% of the raw mark. Given that grade C is usually regarded as the minimum required to continue satisfactorily the study if the subject at A level, is it coincidental that half the candidates nationally achieve grade C or better?

Engineering colleagues bemoan the fact that their students no longer seem to possess a feel for order of magnitude, are unable to carry out simple algebraic manipulation, know very little elementary geometry, are bewildered by relatively simple trigonometric expressions and are able to attempt only the simplest aspects of differentiation. So it is, and one might add that when confronted by a mathematical problem the students will respond with 'can't do it' if the solution requires more than the direct application of a method covered in lectures. The catalogue does not end there, unfortunately; there appears to be very little work ethic among new students, which, for a demanding subject like engineering, could sound the death knell for the degree programmes of today. Certainly, for mathematics at whatever level the only sure recipe for success is practice and more practice.

Tackling the short-term crisis

Does the answer lie in the increased use of computer technology?

At this stage it is worth saying something about the use of computer technology. With the rapid increase in that technology the development of educational software will provide the way forward. There is no longer the need to ask students to solve manually problems which require a great deal of complicated algebraic and numerical manipulation when software packages can deliver the goods. Well, it isn't that simple. Packages such as those designed specifically for finite element analysis, if used carefully are a great boon; if used carelessly then they are a potential danger.

It takes time to learn how to use a package safely; where is that time to be found in a crowded curriculum?

Just as four-figure tables at school were superseded by the slide rule at university, so both have been superseded by the pocket calculator, but with one important difference. It was necessary to estimate the expected answer before carrying out the slide rule calculation and this provided a useful check. Who now estimates an answer before using a pocket calculator, let alone a computer package? As a result, unrealistic answers are quite often obtained, without a flicker of emotion on the part of the perpetrator of the crime of pressing the wrong button or whatever.

How much worse, then, to use a finite element package with no real understanding of what the method is about. Few of us who drive a car really know how the thing works, in any detail. We might be able to carry out the simplest of repairs but anything more complicated and we take the car to a garage for attention. The important line to draw is between what we can successfully accomplish ourselves and what is best taken care of by the garage. But we wouldn't let someone fly an aeroplane without a reasonable understanding of what happens when one of the controls is operated, would we?

How far would those of you who teach structural analysis be prepared to go in the use of educational software? Would civil engineering staff be happy if their students never carried out a manual calculation in a simple case? Would you be satisfied if your students had never designed, built and tested a reinforced concrete beam or a simple steel framework? Why should it be any different for mathematics?

What should be the role for computer technology in the mathematical education of engineering undergraduates? There are two aspects – the use of teaching packages and the use of packages to carry out complicated mathematical analysis. Let us consider the second of these.

No one these days would seriously advocate a return to long division or the manual extraction of square roots. The processes involved did not really advance understanding; rather, they were tedious and off-putting. The graduation to four-figure tables was merely a minor reduction in the tedium. The electromechanical machines which 'speeded up' lengthy calculations were not seen as a threat to understanding and the slide rule was a necessary evil. The danger with the arrival of the pocket calculator was that it was accompanied by a decline in mental arithmetic; even the most trivial of calculations is now carried out on the calculator. This has led to a loss of appreciation for the order of magnitude in the result of a calculation which is about to be performed. This in turn leads to quite nonsensical answers being presented, 'because that's what the calculator said'. A room of 170 cubic metres holding 1400 people has been offered as a serious answer to a problem on determining the room capacity in order that a ventilation system can operate satisfactorily!

The message is therefore, make sure that the computer is used for difficult analysis *after* the basics have been learned manually.

As to the use of computer technology to act as some kind of replacement for the lecture we must treat it as an academic iron pyrites. Experience has shown that even

in the best-equipped Mathematics Learning Resource Centre it is face-to-face human contact which is sought, not a resort to a computer learning package. It is difficult for even the best-designed package to hold a student's attention for too long.

Bring on the sticking plaster

Before we attempt a suggested way to tackle the problem in the medium term it is necessary to put in place a short-term packet of measures to deal with the current situation. One possibility which must be resisted is to offer a crash course in mathematics at the start of the degree programme, or even before it starts. It takes time for the student who is mathematically weak to gain confidence in the subject and competence at it. In the same way there is little point in trying to run a remedial programme of tutorials/lectures alongside the mainstream course: the student will struggle to straddle two horses. What is needed is a recognition that mathematically weak students need to have their enthusiasm for engineering maintained, not killed off by being exposed to mathematics which they cannot understand and which presents a barrier to their progress in the other engineering subjects. This is not to argue that mathematics should not feature as the centrepiece of their studies. Far from it. What is required is time to let the mathematical abilities of the student develop at a sensible rate. The other engineering subjects can be presented in a way which excites the students without placing a heavy mathematical demand on them at the outset.

A book which merits close scrutiny is *The Elements of Structure*, written by Williams and more recently revised by Buckle.⁵ The reader is taken from the situation of a person sitting on a chair to the design of St Paul's Cathedral, with the imposition of a minimal amount of mathematics. Here, surely, is a book to stimulate the interest of the first-year student. There is all too obvious a danger in introducing the freshman to a routine application of standard formulae before he has had an opportunity to obtain a 'feel' for the subject, and any sense of academic curiosity and excitement is probably lost for ever. Once that curiosity has been stimulated then mathematical techniques can be imported to analyse in depth the structures already considered. This will actually help to make the mathematics appear more relevant by showing how understanding can be enhanced by its application to engineering problems.

If this approach is possible for the analysis of structures then it should be possible for fluid mechanics and for other civil engineering subjects. This would give the mathematics lecturer the necessary breathing space to develop some basic mathematics in a structured way so as to build student confidence in, and competence at, applying mathematics to engineering problems. In the first semester the basic mathematics which should have been covered at 'A'-level can be consolidated alongside the other engineering subjects which are treated in a more qualitative way. Then, in the second semester the mathematics module could start where it used to be a few years ago and the other engineering subjects given a more quantitative approach.

An engineering course which has three mathematics modules, say A, B and C, in the first semesters could now have a basic module in Semester 1, followed by

modules A, B and C in Semesters 2, 3 and 4. This means that one of the Year 2 or Year 3 engineering modules would have to be removed or placed on a list of optional modules. Progression would still be based on the modules studied in any year. The student would have more confidence in using mathematics in other modules and would have covered the appropriate mathematics by the time it was required for applications. The alternative is to bumble along as at present, with students understanding progressively less of the quantitative aspects of their subject.

The middle-term 'solution'

Whilst we might wish to see a strengthening of 'A'-level mathematics we must remember that future engineering undergraduates form a minority of the candidature. It is unlikely that the engineering lobby will have much influence in driving through any substantial changes. There is not much likelihood of 'beefing up' Advanced GNVQ mathematics without a wholesale re-structuring of the whole approach. Any successful reforms would, in any case, need to start at primary school level and would take thirteen years to come to fruition. Something radical, therefore, must be done in the meantime. The danger is that we fall into the Danegeld trap, a trap from which it is difficult to escape. This trap is to dilute further the mathematical content of the degree programmes, leading to a non-analytical approach to engineering.

Clearly we need to bring in some 'big guns'. Bodies such as the Engineering Council, the Royal Academy of Engineering and the Engineering Employers' Federation are aware of the problem and are sympathetic to attempts being made to address them. They may have more clout with Government than have the heads of departments of mathematical sciences and the Engineering Professors' Council. The time has arrived when we need to be more politically aware in order to press our case successfully.

Where do we go from here?

For the short term it is suggested that teams comprising both engineering and mathematics staff draw up mathematics and other engineering syllabuses which allow the mathematics modules to get a head start before the other engineering modules make use of that mathematics. This will allow the students to gain that all-important ingredient, confidence, in mathematics while becoming enthused about the other engineering subjects. Then these subjects can be revisited with the necessary mathematics in place to allow a more analytical study. In this way the student can have a more meaningful education, which is surely the aim we all share.

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