Thermal engineering design project: a calorimeter that measures the specific heat of aluminum – part 2

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Abstract A thermal engineering design project requiring the design, construction, and operation of a calorimeter that measures the specific heat of aluminum was assigned to a class of third-year mechanical engineering students. In a previous *IJMEE* paper, the same project was described, along with the author's electrically heated calorimeter that was constructed before the assignment was made. In the current paper, a completely different calorimeter design is described. The current calorimeter is a hollow aluminum cylinder fitted with a plug forming a watertight cavity. The cylinder is heated and allowed to cool in still air. Two tests were performed: one with water in the cavity and one without water in the cavity. The different cooling rates from the two tests allowed computation of the aluminum's specific heat. A class of junior (third-year) mechanical engineering students, working in teams, produced designs using electrically heated aluminum samples and calorimeters that mixed aluminum and water at initially different temperatures. The 16 student groups plus the author produced 113 data points with a mean specific heat value that deviated by 7.8% from a published value.

Keywords calorimeter; specific heat; thermal design

Notation

A	surface area, m^2
Bi	Biot number, dimensionless
c	specific heat, kJ/(kg K)
c_1, c_2	constants
d	diameter, m
g	acceleration due to gravity, 9.81 m/s^2
$\bar{h}$	heat loss coefficient, $W/(m^2 K)$
m	mass, kg
$\overline{Nu}$	Nusselt number, dimensionless
q	heat transfer rate, W
Ra	Rayleigh number, dimensionless
T	temperature, °C
t	time, s
U	internal energy, J

Greek symbols

α	thermal diffusivity, m^2/s
β	coefficient of volumetric thermal expansion, K^{-1}
ν	kinematic viscosity, m^2/s
θ	$(T - T_\infty)$, °C

Subscripts

Al	aluminum
a	calorimeter with no water
b	calorimeter with water
d	diameter, m
H_2O	water
∞	ambient air
1	state 1
2	state 2

Introduction

A thermal design project requiring design, construction and operation of a calorimeter that measures the specific heat of aluminum was assigned to third-year mechanical engineering students during spring quarter 2001. In a previous *IJMEE* paper [1] the same project, with results, was described in detail. In this paper a completely different calorimeter is described, along with the results of the latest class.

Joseph Black [2] is credited with the discovery of specific heat, based on experiments that he performed in 1760. Black mixed water and mercury at different temperatures and ended up with a final temperature that surprised him. This 'method of mixtures' was described to my class as well as the general arrangement of an electric heater calorimeter (also know as a Nernst-type calorimeter). However, specific design details and testing methods were not provided because this was part of the creative design left to the students. I first developed a design that produced good results before the project was assigned. However, this design was not disclosed until the last day of class. My design is given first, followed by the students' designs, and then the collective results.

Author's design

All calorimeters involve either the heating or cooling of sample materials, which results in an unavoidable heat exchange (usually a heat loss) with the surroundings. To minimize this heat loss, workers [3] have produced designs that have included vacuum environments and guard heaters. An alternative approach is neither to minimize nor to calculate the unavoidable heat loss but to eliminate it *in the analysis*. This is accomplished with two material samples with identical exteriors and identical surroundings but with different interiors. When the exterior temperatures of both samples are the same, then the unknown heat loss to the surroundings is also the same – this allows cancellation of the heat loss in the analysis. This is the approach I took with my electric calorimeter described in the first IJMEE paper [1]. It is also the approach I took with my current horizontal cylinder calorimeter.

In the current design I use an aluminum cylinder bored out to produce a hollow interior. There is a single open end that is fitted with an aluminum, pipe-thread plug that provides a watertight interior cavity. Fig. 1 shows this cylinder. The cylinder is suspended horizontally by two thin wires. It has an attached thermocouple and there

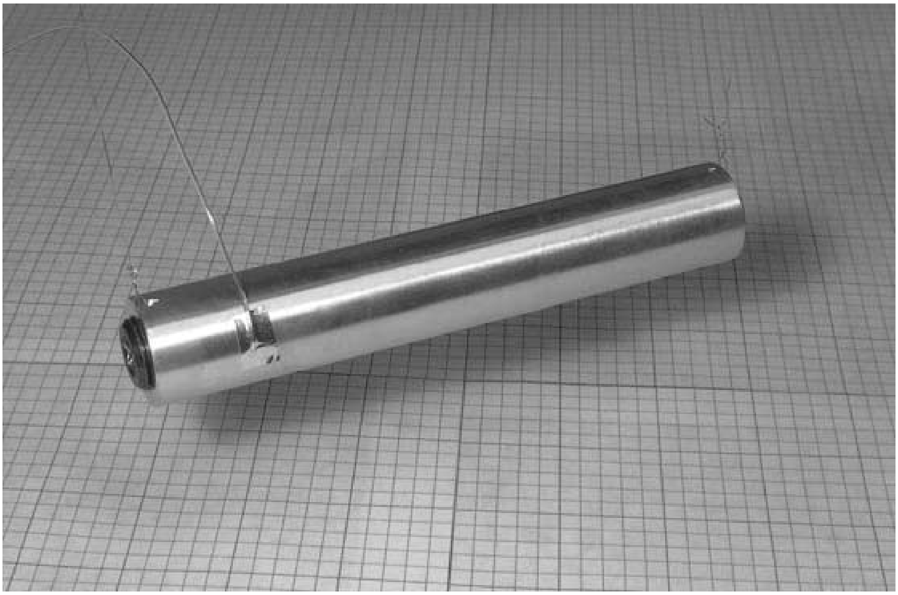

Fig. 1 *The horizontal cylinder calorimeter suspended by two thin wires. A single 30-gage, type K thermocouple is attached on the exterior surface with a piece of aluminum tape.*

is a second thermocouple to measure ambient air temperature. The cylinder is heated with a hot-air gun* and then allowed to cool in still air under two conditions: first with no water in the interior cavity and then with water. Each run produces a unique cooling curve. These cooling curves are used to compute specific heat of aluminum using the specific heat of water as a reference.

The current design evolved from two earlier designs. In all of these designs the idea was to cancel the unknown heat loss from two calorimeters with identical exteriors and surroundings but with different interiors. In the first design, two identical Styrofoam cups were used. One cup contained hot water and the second cup contained hot water and aluminum. This calorimeter is shown in Fig. 2. The different cooling rates that resulted due to the different interiors were used in the analysis to compute the specific heat of the aluminum – however, the results were 50% too high. Subsequent thermocouple temperature measurements revealed a temperature difference of about 1.5 °C between the upper and lower levels in the water. However, the analysis calls for a uniform temperature, so this temperature stratification is believed to have contributed to the error.

A second design was developed to eliminate this temperature stratification. In this design an aluminum plate was drilled with multiple holes that were filled with water

* Caution must be taken to restrict heating to below 100 °C. The cylinder with water and plug is a potential steam pressure vessel if heated above 100 °C.

Fig. 2 *The Styrofoam cup calorimeter. The cup on the left normally holds the aluminum core that has been removed and placed in front of it. This core is constructed from 184 aluminum rods, 2.38 mm in diameter and 60 mm long. The rods are held with a circular plate drilled to keep them separated. Water just covers the top of the plate when the core is in the cup. The cup on the right is filled only with water. Both cups have the same height of water and are closed with plastic lids. Each cup is instrumented with two thermocouples, one near the surface of the water and one near the bottom.*

(see Fig. 3). In this second design, the water has a shallower depth and there is more aluminum compared with water – both features that will favor temperature uniformity. The vertical depth of the holes was about 20 mm, which was much less than the 70 mm water depth in the Styrofoam cups. Additionally, there was a 5 : 1 aluminum-to-water ratio in the aluminum plate calorimeter, compared with a 1.8 : 1 ratio in the Styrofoam cup calorimeter. A thermocouple attached to the outer aluminum surface and a thermocouple inserted in one water hole in the aluminum plate calorimeter indicated uniform temperature throughout all of the testing. However, the problem with the aluminum plate calorimeter was evaporation. Even with a thin aluminum plate covering the holes and, later, aluminum tape further sealing the cover, evaporation was unavoidable. The computed specific heat values were consequently about 15% too low.

The third design (the horizontal cylinder calorimeter described initially) incorporated features to eliminate the problems discovered in the first two designs. The horizontal cylinder calorimeter has a watertight chamber, 14 mm water vertical depth, and a 7.4 : 1 aluminum-to-water mass ratio. The combination of these features ensured no evaporation loss, and uniform temperatures were produced.

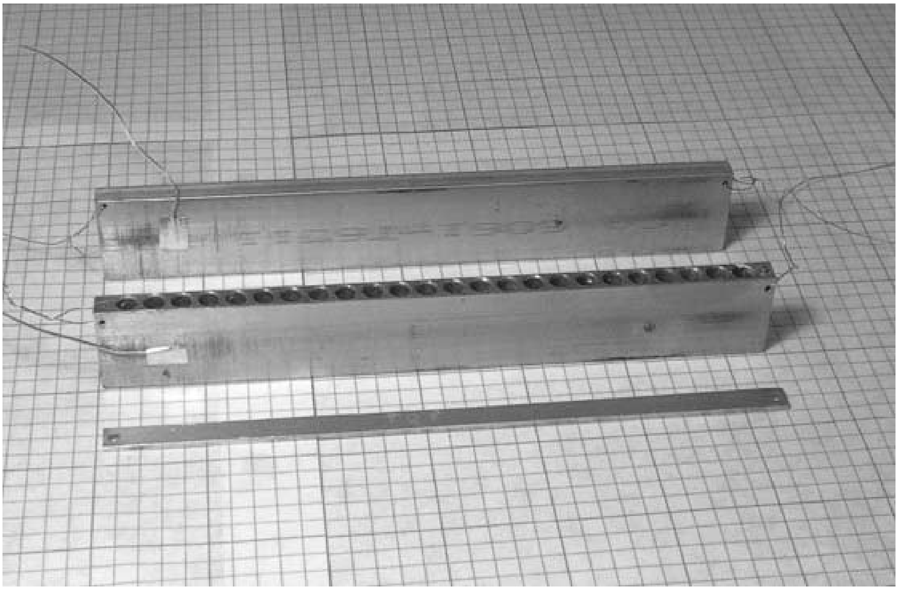

Fig. 3 *The aluminum plate calorimeter. The plate in the background is solid aluminum, 162 mm long, 25.4 mm high, and 6.35 mm thick. The plate in the foreground has 24 drilled holes, 4.76 mm in diameter and 20 mm deep. The thin aluminum strip ahead of this plate covers the holes. The exterior dimensions of the drilled plate with lid are identical to the solid plate. Both plates are suspended horizontally as they appear in the photo, by thin wires. Each plate has a type K, 30-gage thermocouple attached on its exterior surface with a piece of aluminum tape.*

Experiments

The dimensions of the horizontal cylinder calorimeter are: 148.5 mm long, 25.4 mm outside diameter, 19.1 mm inside diameter, interior chamber length (with the plug in place) 135 mm, and mass (with the aluminum plug) 142.7 g.

The cylinder was suspended horizontally by fine wires and heated with a hot-air gun to about 70 °C and then allowed to cool in still air, which was about 20 °C. Two runs were made: one with no water in the interior chamber and one with water. Furthermore, two sets of runs were performed: one with 19.3 g of water and one with 20.0 g of water. A PC-based data acquisition system was used to record cylinder wall temperature and ambient air temperature every 10 seconds. The cooling curves from the first set of runs appear in Fig. 4. In Fig. 4, $(T - T_\infty)_a$ represents the cylinder cooling without water and $(T - T_\infty)_b$ represents the cylinder cooling with water. Both curves were set on the time axis with $(T - T_\infty)_a = (T - T_\infty)_b = 51.4$ °C at $t = 0$.

In the analysis section below, it is necessary to have the derivatives of these two cooling curves at $t = 0$. Also, it is convenient to make the substitution $\theta = (T - T_\infty)$. First, we fit polynomials to these curves:

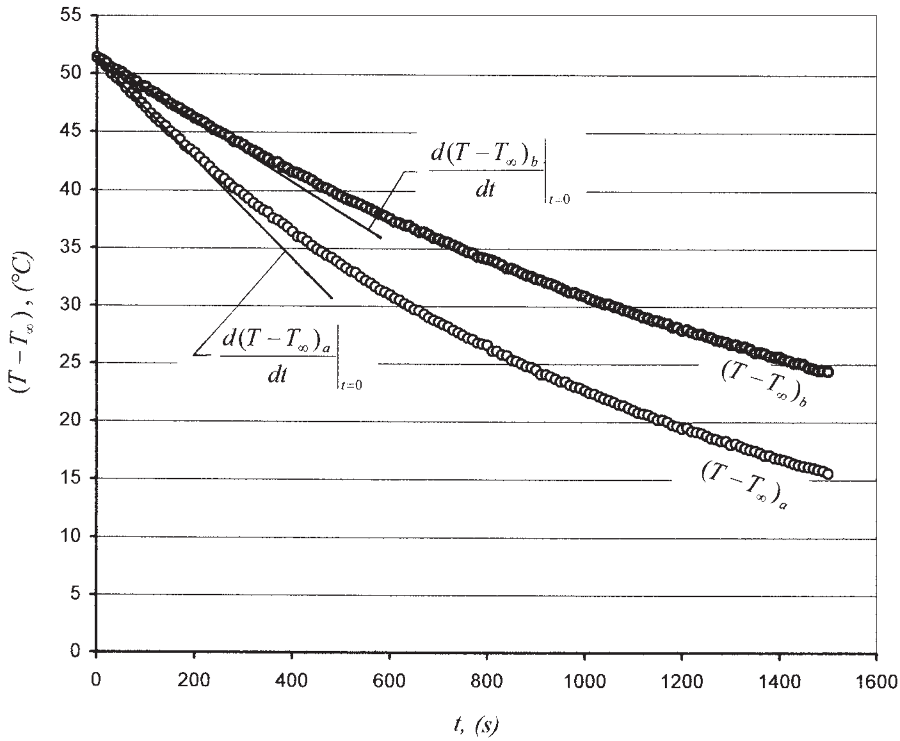

Fig. 4 *The cooling curves for the horizontal cylinder calorimeter. The* $(T - T_\infty)_a$ *curve is for the cylinder with no water and the* $(T - T_\infty)_b$ *curve is for the cylinder with water.*

$$\theta_a = (T - T_\infty)_a = 1.606 \times 10^{-12} t^4 - 9.049 \times 10^{-9} t^3 + 2.479 \times 10^{-5} t^2 - 4.612 \times 10^{-2} t + 51.45 \quad (1)$$

$$\theta_b = (T - T_\infty)_b = 2.993 \times 10^{-13} t^4 - 2.251 \times 10^{-9} t^3 + 9.200 \times 10^{-6} t^2 - 2.781 \times 10^{-2} t + 51.43 \quad (2)$$

The correlation coefficient for θ_a is 0.999965 and the correlation coefficient for θ_b is 0.999940. The derivatives for these two curves, evaluated at $t = 0$, become:

$$\left.\frac{d\theta_a}{dt}\right|_{t=0} = -4.612 \times 10^{-2}\ °C/s \quad (3)$$

$$\left.\frac{d\theta_b}{dt}\right|_{t=0} = -2.781 \times 10^{-2}\ °C/s \quad (4)$$

Analysis

A lumped heat capacity* analysis is used for the horizontal cylinder calorimeter cooling in still air first without water and then with water. The first law of thermodynamics applied to the cylinder, $-q = \frac{\mathrm{d}U}{\mathrm{d}t}$, is written for cooling without water (subscript 'a') and with water (subscript 'b'):

$$-\bar{h}_{\mathrm{a}} A \theta_{\mathrm{a}} = m_{\mathrm{Al}} c_{\mathrm{Al}} \frac{\mathrm{d}T_{\mathrm{a}}}{\mathrm{d}t} \tag{5}$$

$$-\bar{h}_{\mathrm{b}} A \theta_{\mathrm{b}} = \left(m_{\mathrm{Al}} c_{\mathrm{Al}} + m_{\mathrm{H_2O}} c_{\mathrm{H_2O}}\right) \frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}t} \tag{6}$$

Equations (5) and (6) are solved using two methods: in the first, the derivatives are evaluated directly; and in the second, the equations are integrated.

Analysis using the derivatives

At $t = 0$, $\theta_{\mathrm{a}} = \theta_{\mathrm{b}}$ and $\bar{h}_{\mathrm{a}} = \bar{h}_{\mathrm{b}}$. Furthermore $T_{\infty,\mathrm{a}}$ and $T_{\infty,\mathrm{b}}$ remain essentially constant during the 1500 s cooling period, so $\frac{\mathrm{d}T_{\mathrm{a}}}{\mathrm{d}t} \cong \frac{\mathrm{d}\theta_{\mathrm{a}}}{\mathrm{d}t}$ and $\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}t} \cong \frac{\mathrm{d}\theta_{\mathrm{b}}}{\mathrm{d}t}$. Using this information, equations 5 and 6 reduce to:

$$c_{\mathrm{Al}} = c_{\mathrm{H_2O}} = \frac{m_{\mathrm{H_2O}}}{m_{\mathrm{Al}}} \left[\frac{\left.\frac{\mathrm{d}\theta_{\mathrm{a}}}{\mathrm{d}t}\right|_{t=0}}{\left.\frac{\mathrm{d}\theta_{\mathrm{b}}}{\mathrm{d}t}\right|_{t=0}} - 1 \right]^{-1} \tag{7}$$

where the derivatives are given by equations 3 and 4.

Analysis using integration

The $\bar{h}$ notation in equations 5 and 6 represents a heat loss coefficient that combines both convection and radiation. However, convection is the dominant mode because the cylinder is polished aluminum with a low emissivity and thus there is low thermal radiation. Furthermore the natural convection is primarily laminar, so the well known $\overline{Nu}_d = \mathrm{c}_1 Ra_d^{1/4}$ relation may be used, where $\overline{Nu}_d = \frac{\bar{h} d}{k}$, c_1 is a constant, and $Ra_d = \frac{g\beta(T - T_\infty)d^3}{\nu\alpha}$. This directly reduces to:

$$\bar{h} = \mathrm{c}_2 (T - T_\infty)^{1/4} \tag{8}$$

where $\mathrm{c}_2 = \mathrm{c}_1 k \left[\frac{g\beta}{\nu\alpha d}\right]^{1/4}$.

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* The lumped heat capacity analysis requires uniform temperature at any instant of time – see, for example, Bejan [4]. To ensure uniform temperature the Biot number, $Bi \leq 0.1$. For the aluminum cylinder with no water, $Bi \approx 0.003$. For the aluminum cylinder with water, $Bi \approx 0.05$.

TABLE 1 *Final results*

Run set	$\frac{m_{H_2O}}{m_{Al}}$	$\theta_{a,1}$ (°C)	$\theta_{a,2}$ (°C)	$\theta_{b,1}$ (°C)	$\theta_{b,2}$ (°C)	c_{Al} (kJ/(kg K)): analysis using derivatives (equation 7)	c_{Al} (kJ/(kg K)): analysis using integration (equation 11)	c_{Al} (kJ/(kg K)): analysis using integration (equation 12)
1	0.1352	51.4	15.6	51.4	24.4	0.858	0.812	0.832
2	0.1402	48.5	15.6	48.5	23.3	0.886	0.930	0.932

We use equation 8 in equations 5 and 6, make the substitution $\theta = (T - T_\infty)$, and note that T_∞ remained very stable (constant) during the cooling processes. The result is:

$$-c_2 A\theta_a^{5/4} = m_{Al}c_{Al}\frac{d\theta_a}{dt} \tag{9}$$

and

$$-c_2 A\theta_b^{5/4} = \left(m_{Al}c_{Al} + m_{H_2O}c_{H_2O}\right)\frac{d\theta_b}{dt} \tag{10}$$

Equations 9 and 10 are separated, integrated, and combined to give:

$$c_{Al} = c_{H_2O} = \frac{m_{H_2O}}{m_{Al}}\left[\frac{\theta_{a,2}^{-1/4} - \theta_{a,1}^{-1/4}}{\theta_{b,2}^{-1/4} - \theta_{b,1}^{-1/4}} - 1\right]^{-1} \tag{11}$$

A second integral solution may be performed. Equation 1 may be used for θ_a and equation 2 for θ_b. Using these substitutions, equations 9 and 10 are separated, integrated, and combined to give:

$$c_{Al} = c_{H_2O} = \frac{m_{H_2O}}{m_{Al}}\left[\frac{(\theta_2 - \theta_1)_a}{(\theta_2 - \theta_1)_b}\frac{\int_0^{1500s}\theta_b^{5/4}dt}{\int_0^{1500s}\theta_a^{5/4}dt} - 1\right]^{-1} \tag{12}$$

Equations 7, 11, and 12 represent three different ways of reducing the same data to obtain the specific heat of aluminum.

Results

Two sets of runs were performed. The results are given in Table 1. The first set of runs is shown in Fig. 4. The cooling curves in this figure were fitted with the polynomials given by equations 1 and 2. The derivatives of these polynomials at $t = 0$ are given by equations 3 and 4. Using these data, plus $c_{H_2O} = 4.18$ kJ/(kg K) as a reference, equation 7 produced $c_{Al} = 0.858$ kJ/(kg K). Equations 11 and 12 were also used to obtain c_{Al}. All of the final results plus selected experimental data appear in

Table 1. In this table the second column gives the mass ratio of the water to the aluminum cylinder. The third, fourth, fifth, and sixth columns give the temperature differentials at the beginning (subscript 1) and at the end (subscript 2) of the runs for the cylinder without water (subscript a) and with water (subscript b). The last three columns give c_{Al} as computed by equations 7, 11, and 12.

The mean value of the specific heats shown in Table 1 is 0.875 kJ/(kg K), with a standard deviation of 0.050 kJ/(kg K). So, the final result may be expressed as $\bar{c}_{Al}$ = 0.875 kJ/(kg K) ± 5.7%. Comparison with published specific heat values was imprecise because the exact aluminum alloy used for the calorimeter was unknown. The aluminum rod, already at the finish length, was obtained from a scrap bin. There were no markings indicating the alloy. However, one common aluminum alloy used for sheets and rods is 2024. The specific heat of this aluminum alloy is 0.895 kJ/(kg K). For pure aluminum, c_{Al} = 0.916 kJ/(kg K). Both of these specific heat values are at 55 °C, the mean temperature for both sets of runs. Using the 2024 alloy as a reference, the final experimental result is 2.2% too low.

Students' designs

Two sections of third-year mechanical engineering students were assigned into 16 groups with four or three students per group. Twelve groups designed equipment using the method of mixtures and four groups used electrically heated aluminum samples. In the method of mixtures, aluminum and water, initially at different temperatures, are mixed together and a final equilibrium temperature is obtained. In the electrically heated samples the aluminum is insulated and electrical power input is measured. The major challenge for both designs was to minimize heat loss or properly account for it. Styrofoam and vacuum containers minimized but did not eliminate heat loss. To account for the unavoidable heat loss the students recorded the temperatures during the electric heating or the initial mixing of water and aluminum and also during cooling of the calorimeters. Then they used this cooling curve in a variety of ways to account for heat loss that occurred during heating or mixing. Some groups used the cooling curve to determine an instantaneous cooling rate by taking the derivative. Other groups evaluated the heat loss over an interval by integration.

Collective results

The 16 student groups made multiple runs once they got their equipment operational. There were 113 data points, including the author's work. All of the data are shown in Fig. 5. The symbols in this figure distinguish between the method of mixtures and the method using electric heater calorimeters. The mean specific heat value from the 113 data points is 0.953 kJ/(kg K) with a standard deviation of 0.241 kJ/(kg K) or 25.3%. Most of the aluminum used was of an unknown alloy. Two reference lines are plotted in Fig. 5: pure aluminum and aluminum alloy 2024. With reference to the alloy, the final collective result is 7.8% too high.

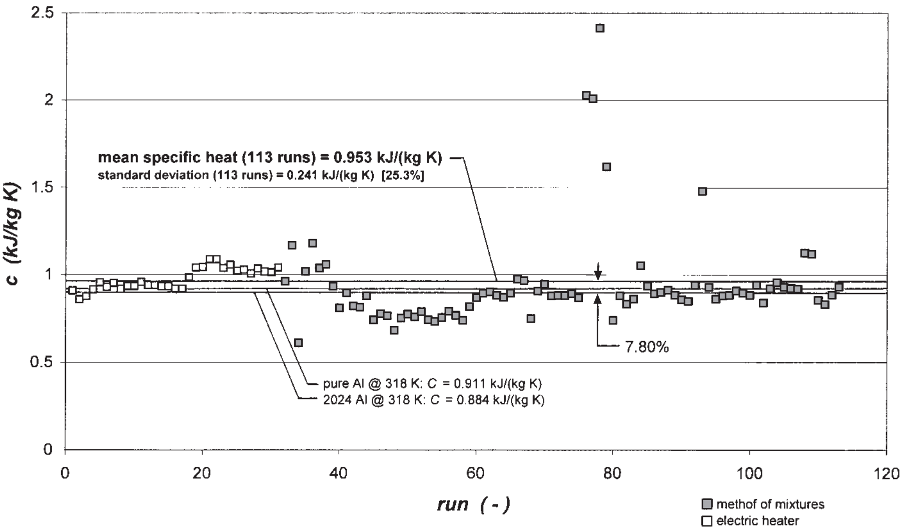

Fig. 5 *Specific heat of aluminum. Collective results from 16 student groups plus the author.*

Conclusions

The specific heat of aluminum was experimentally obtained by equipment of three different designs. In the first design, the author continuously measured the temperature of a hollow aluminum cylinder as it cooled in still air under two conditions: first with no water in the interior cavity and then with water. The temperature data were reduced using three different methods and produced a mean specific heat value that was 2.2% below a published value. In the second design some students used an aluminum sample, insulated and electrically heated. In the third design some other students mixed aluminum and water, initially at different temperatures, and then obtained a final equilibrium temperature. The final collective results were 7.8% higher than a published specific heat value. An important part of this design project is that the final result can be qualitatively compared with the published specific heat of aluminum, which provides a performance benchmark.

Acknowledgement

The author wishes to thank the 62 students in both sections of ME 344 Thermal Engineering, spring quarter 2001, for their enthusiasm and productivity, which resulted in a successful assignment.

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