Utilization of the PSPICE code for the unsteady thermal response of composite walls in a heat transfer course

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Abstract This paper on engineering education introduces a robust computational technique named the network simulation method (NSM) to undergraduate students on heat transfer courses for the simultaneous determination of temperature–time histories and heat flux–time histories inside layered walls easily, quickly, and accurately. NSM relies on the existing physical analogy between unsteady unidirectional conduction of heat and unsteady-state flow of electric current, the RC analogy. This unsteady analogy is a natural extension of the steady analogy that holds between unidirectional heat conduction and the flow of electric current, the R analogy. The implementation of NSM is carried out with the thermal analysis of a composite wall of the nozzle of an experimental rocket engine during a ground firing test. The composite wall of the nozzle comprises a metallic substrate and a ceramic coating. The NSM-based numerical simulations have been performed with the commercial code PSPICE.

Keywords resistance capacitance analogy; nozzle wall; melting temperature; thermal barrier coating

Notation

c	specific isocoric heat capacity, J/kg.K
C_i	capacitance of compartment i, F
h_c	convective heat transfer coefficient, $W/m^2.K$
h_r	radiative heat transfer coefficient, $W/m^2.K$
h_t	total heat transfer coefficient, $W/m^2.K$
j	heat flux density, W/m^2
k	thermal conductivity, W/m.K
L	thickness of composite wall, $L_1 + L_2$, m
L_1	thickness of ceramic coating, m
L_2	thickness of metallic substrate, m
N	number of cells
R_i	resistance of compartment i, ohm
t	time, s
T	temperature, K
T_0	initial temperature, K
$T_{1,2}$	temperature at the interface of the ceramic coating and the metallic substrate, K

T_∞ mean bulk temperature, K
x space variable, m

Greek letters
α thermal diffusivity, $k/\rho c$, m^2/s
ρ density, kg/m^3

Subscripts
1 ceramic coating
2 metallic substrate
mp melting point

Introduction

All textbooks on basic heat transfer, without exception, are saturated with the electric circuit analogy for the treatment of steady, unidirectional conduction of heat in layered walls, the R analogy. In contrast, all textbooks are deficient in the utilization of the electric circuit analogy for the equally or more important unsteady, unidirectional conduction of heat in layered walls (see, for instance, Incropera and DeWitt [1], Thomas [2], Mills [3], Kreith and Bohn [4]). Actually, there are many notable apparatusses for heating/cooling operations in industry that are constructed with layered walls, such as HVAC systems, fire protection devices, gas turbines, rocket engines, and nuclear reactors.

For the examination of unsteady, unidirectional heat conduction in single walls of large plates, long cylinders and spheres exposed to gaseous or liquid convective environments, all textbooks briefly delineate the method of separation-of-variables as the primary technique [1–4]. This elegant method leads to analytic solutions that are represented by infinite Fourier series for rectangular and spherical coordinate systems and infinite Fourier–Bessel series for the cylindrical coordinate system. These analytic solutions are capable of predicting (a) the unsteady temperature distributions in these regular bodies and (b) the total heat transfer between these regular bodies and a surrounding fluid. On the positive side, the striking characteristic of these infinite series for purposes of numerical evaluation is their rapid convergence for very long times, necessitating only one term that embodies one eigenvalue. On the negative side, the infinite series diverge markedly for short times, and many terms need to be retained to secure adequate accuracy.

In general, the majority of problems on unsteady, unidirectional conduction of heat in composite walls are more complicated than their counterparts in single walls and have been traditionally treated with complex analytical methods [5, 6] or finite-difference methods [6]. First, obtaining analytical solutions demands intricate algebraic operations to calculate the eigenvalues and more importantly to match the interface temperatures and the interface heat fluxes at short times. Second, although finite-difference methods overcome the obstacles inherent in the analytical methods, their implementation requires approximate finite-difference formulations to represent both the time and the spatial temperature derivatives in each material.

Obviously, these two computational avenues are beyond the scope of undergraduate students on heat transfer courses [1–4].

In view of the foregoing, the central objective of this paper on engineering education is to provide undergraduate students on heat transfer courses with a robust computational technique that enables them to determine simultaneously temperature–time histories and heat flux–time histories inside layered plates, layered cylin-

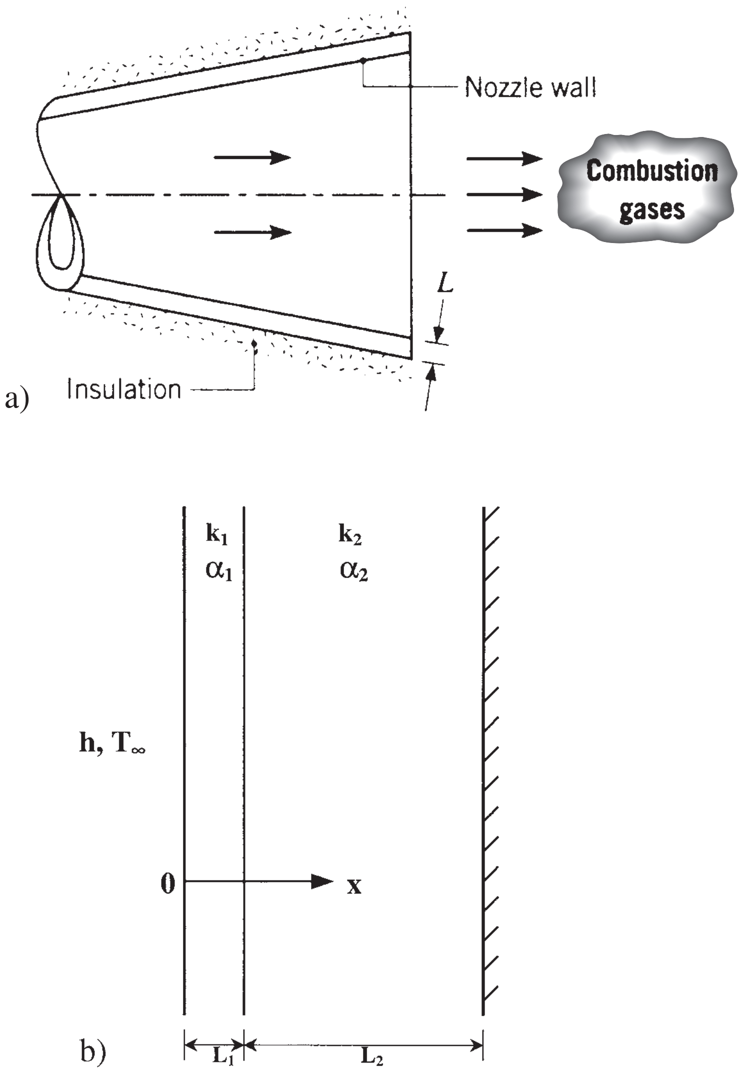

Fig. 1 *(a) View of the cross-section of the composite nozzle wall, (b) the computational domain.*

ders and layered spheres, easily, quickly, and accurately. To achieve this goal, the paper focuses on the existing analogy between unsteady, unidirectional heat conduction and unsteady flow of electric current, the RC analogy. Indeed, this analogy is the natural extension of the standard R analogy. The main feature of the (unsteady) RC analogy arises from the fact that it uses discrete intervals of space and real, continuous time as the independent variable.

A two-layer composite wall

A sketch of the cross-section of a nozzle wall of an experimental rocket engine is shown in Fig. 1a. The composite annular wall consists of two concentric layers of materials. The first layer is a thin ceramic coating ('thermal barrier') and the second layer is a thick metallic substrate (structural element); the latter is wrapped by a layer of insulation.

One feasible approach for achieving high operating temperatures in the combustor of a rocket engine without damaging the structural integrity of the nozzle wall involves the application of a plasma-sprayed ceramic coating of zirconia (ZrO_2) onto the surface of the metallic nozzle, which is normally made from stainless steel. Current reviews of plasma-sprayed ceramic coatings on metallic substrates are provided by Lindsay [7] and Upadhya [8].

The nozzle wall is maintained at ambient temperature, T_0, throughout. At a given time, $t = 0$, the ground firing test begins and an intense heat flux is suddenly applied to the exposed surface of the ceramic coating covering the nozzle wall. The incoming heat flux is caused by the turbulent movement of a stream of hot combustion gases emerging from the combustor. The characterization of the combustion gases requires a mean bulk temperature, T_∞, and a total heat transfer coefficient, $h_t = h_c + h_r$, that accounts for turbulent forced convection and radiation transfer.

The two key assumptions in the preparation of the model are: (a) the thickness-to-diameter ratio of the composite nozzle wall is small so that the wall may be safely approximated by a flat plate in a rectangular coordinate system, (b) there is perfect contact between the coating and the substrate, so that the thermal contact resistance is insignificant.

As indicated in Fig. 1b, the origin of the coordinate system is taken at the exposed surface of the ceramic coating for convenience. Accordingly, the one-dimensional heat conduction equation for the ceramic coating is

$$\rho_1 c_1 \frac{\partial T_1}{\partial t} = k_1 \frac{\partial^2 T_1}{\partial x^2}, \quad 0 < x < L_1 \tag{1}$$

and the one dimensional heat conduction equation for the metallic substrate is

$$\rho_2 c_2 \frac{\partial T_2}{\partial t} = k_2 \frac{\partial^2 T_2}{\partial x^2}, \quad L_1 < x < L_1 + L_2 \tag{2}$$

The initial conditions are:

$$T_1 = T_2 = T_0, \quad t = 0 \tag{3}$$

and the boundary conditions are

$$h(T_\infty - T_1) = -k_1 \frac{\partial T_1}{\partial x}, \quad x = 0 \tag{4}$$

$$T_1 = T_2, \quad x = L_1 \tag{5}$$

$$k_1 \frac{\partial T_1}{\partial x} = k_2 \frac{\partial T_2}{\partial x}, \quad x = L_1 \tag{6}$$

$$\frac{\partial T_2}{\partial x} = 0, \quad x = L_1 + L_2 \tag{7}$$

Note that equations (5) and (6) represent the temperature continuity and the heat flux continuity at the coating–substrate interface.

Network simulation method

The network simulation method (NSM) is a numerical procedure that has its foundation in the physical analogy between unsteady, unidirectional heat conduction and unsteady flow of electric current, the RC analogy. From a historical perspective, it should be mentioned that the RC analogy was successfully employed by Paschkis and Baker [9] for solving the linear unidirectional heat conduction equation in a plane wall using an analog computer. An analog computer is made of an electric network composed of resistances and capacitors that were carefully selected to duplicate real thermal systems. The authors named the analog computer a heat and mass flow thermal analyzer. In the spirit of the RC analogy, the voltage differences and instantaneous currents corresponding to temperature differences and instantaneous heat transfer respectively have been measured [9].

The implementation of the NSM commences with the partition of the physical domain $0 < x < L$ into a finite number of cells of equal thickness, Δx, creating a computational domain.

In a typical compartment, i, the energy balance of a cell may be expressed as:

$$\Delta x(\rho c)_i \frac{\mathrm{d}T_i}{\mathrm{d}t} = -\frac{\Delta j}{\Delta x} = \frac{j_{i-\Delta} - j_{i+\Delta}}{\Delta x} \tag{8}$$

where $j_{i-\Delta}$ and $j_{i+\Delta}$ signify the respective heat flux densities entering and leaving compartment i, $i = 0, 1, 2 \ldots, N$, as shown in Fig. 2a. In turn, the heat flux densities, $j_{i-\Delta}$ and $j_{i+\Delta}$, are defined by Fourier's law as:

$$j_{i\pm\Delta} = \pm k \frac{T_i - T_{i\pm\Delta}}{\Delta x/2} \tag{9}$$

In this generic equation, $T_{i-\Delta}$, T_i and $T_{i+\Delta}$ denote the temperatures at the left extreme, at the center and at the right extreme of compartment i, respectively. Hence, the combination of equations (8) and (9) gives rise to the following differential-difference equation:

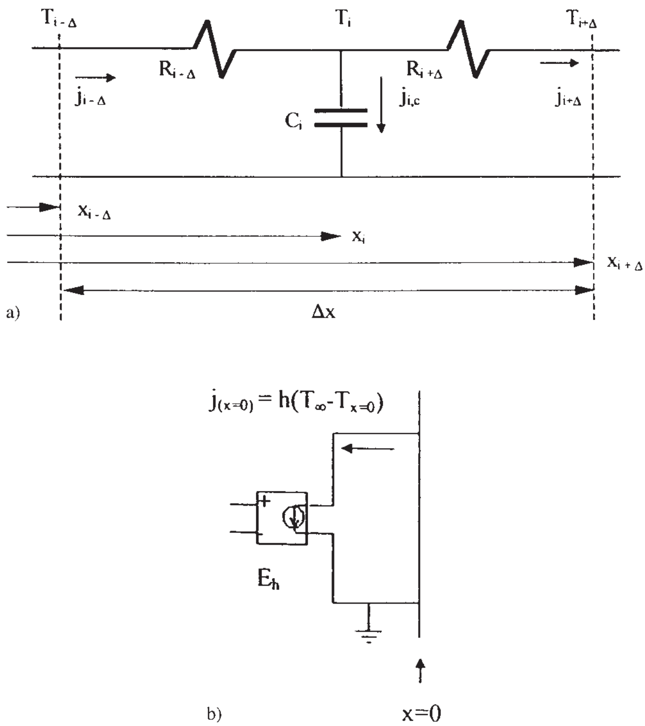

Fig. 2 *Network model for: (a) a typical interior cell and (b) the two boundary cells.*

$$\Delta x(\rho c)_i \frac{\mathrm{d}T_i}{\mathrm{d}t} = k\frac{T_{i-\Delta} - T_i}{\Delta x/2} - k\frac{T_i - T_{i+\Delta}}{\Delta x/2} \tag{10}$$

where the left side is a differential term, and the two terms of the right side are finite-difference terms. At this point, each term in equation (10) is appropriately redefined as an electric current, leading to:

$$j_{i-\Delta} - j_{i+\Delta} - j_{i,c} = 0 \tag{11}$$

where

$$j_{i,c} = \Delta x(\rho c)_i \frac{\mathrm{d}T_i}{\mathrm{d}t} \tag{12}$$

In the context of the prevalent equivalence, equation (10) or (11) may be interpreted as Kirchhoff's current law (implying energy conservation), where the temperature, T_i, turns out to be a continuous single-valued dependent variable that satisfies Kirchhoff's voltage law in each compartment.

With this preamble, the stage is now set for the treatment of unsteady, unidirectional conduction of heat in a one-side heated, two-material plane wall with the NSM. Recognizing that time is a continuous independent variable, use is made of an equivalent electric network whose characteristic variables are the voltage, V_i (analogous to the temperature, T_i) and the electric current, j_i (analogous to the heat flux density, j_i).

Within the framework of the composite plane wall (a ceramic coating and a metallic substrate) under study here, equation (10) envisions the participation of two equal resistors with resistances

$$R_{i\pm\Delta,1} = \frac{\Delta x}{2k_2}, \quad R_{i\pm\Delta,2} = \frac{\Delta x}{2k_2} \tag{13}$$

and one capacitor of capacitance

$$C_{i,1} = \Delta x\left(\rho_1 c_1\right)_i, \quad C_{i,2} = \Delta x\left(\rho_2 c_2\right)_i \tag{14}$$

which is provided by equation (8). Accordingly, the connection between the two resistors $R_{i-\Delta}$ and $R_{i+\Delta}$ and the capacitor C_i in the trio of interior nodal points $i - 1$, i and $i + 1$ is illustrated in the electric circuit of Fig. 2a. The generalization of these three electric elements in one cell to the total number of N cells which are interconnected in series gives the network model. The total number of cells is given by $N = N_1 + N_2$, distributed in this manner N_1 for the ceramic coating and N_2 for the metallic substrate.

Finally, the remaining steps are reserved for the inclusion of the boundary and initial conditions. These steps are performed with special electric devices. First, a voltage-control current source, G, is connected at the first cell, $i = 0$, to handle the dominant convective boundary condition (equation 4). Second, a resistor of infinite value, R_∞ (an open circuit), is connected to the end of the last cell, $i = N$, to regulate the insulated boundary condition (equation 7). Both boundary conditions are adequately represented in Fig. 2b. Third, the initial condition is adjusted by charging the capacitors of the network model to the magnitude of the initial temperature in equation (3).

In this contemporary era, the estimations of temperatures and heat flux densities do not need to be done indirectly with measurements of voltages and electric currents in a heat and mass flow thermal analyzer anymore. With the advent of digital computers, potent software, and versatile graphing software, these time-consuming operations are a thing of the past. Nowadays, these operations can be simulated numerically quickly and efficiently with a commercial code, PSPICE, which has been tailor-made for personal computers [10]. This code has its origins in the family of codes. SPICE [11], designed by electrical engineers at the University of California at Berkeley with the purpose of investigating the unsteady behavior of complex circuits that contain a variety of elements, such as resistors, capacitors, voltage sources, voltage-control current sources, etc. PSPICE has been tested thoroughly by Alhama [12] for the analysis of a multitude of linear and nonlinear problems of heat conduction.

Advantages of the network simulation method over the finite-difference technique

The NSM has several advantages over the finite-difference technique. One key advantage of NSM is that the numerical errors are quantified in terms of the spatial interval, Δx, exclusively, without the intervention of the time interval, Δt. Thus, the errors depend on Δx solely and can be easily controlled. Another distinctive advantage of the NSM over standard finite-difference techniques is that, with the former, the temperature field and the heat flux density field are computed immediately, without any a posteriori numerical differentiation of the temperature field. An unparalleled advantage of the NSM is its immediate suitability to composite plane walls, because the boundary conditions of temperature continuity and heat flux continuity across the material interfaces (equation 6) are automatically satisfied in an electric sense. Both are regulated automatically because of the satisfaction of the current conservation law in the electric circuits of the network.

Practical example

The nozzle wall of a certain experimental rocket engine has to be tested in a ground firing test for purposes of research and development. The metallic wall has a thickness $L_2 = 25\,\text{mm}$ and is made from stainless steel AISI 304. For added protection, a coating of thickness $L_1 = 10\,\text{mm}$ made of zirconia (ZrO_2) is sprayed onto the inner surface of the stainless steel wall, creating a 'thermal barrier'. The outer surface of the nozzle wall is wrapped with a thick layer of insulation, so that the heat losses to the ambient air are minimal.

Because of safety reasons, the rocket industry has established that ground firing tests of experimental rocket engines must be of short duration. As the firing test begins, the inner surface of the zirconia is suddenly exposed to a stream of hot turbulent combustion gases characterized by a bulk temperature $T_\infty = 2300\,\text{K}$ and a total heat transfer coefficient $h_t = 5000\,\text{W/m}^2.\text{K}$. Prior to the firing, the temperature of the rocket engine is in equilibrium with the temperature of the atmospheric air at $T_i = 25\,°\text{C}$.

The averaged thermophysical properties of the stainless steel AISI 304 are taken from Touloukian and Ho [13]: $\rho_2 = 7900\,\text{kg/m}^3$, $c_2 = 580\,\text{J/kg.K}$, $k_2 = 25\,\text{W/m.K}$ and $\alpha_2 = 5.5 \times 10^{-6}\,\text{m}^2/\text{s}$. The melting point of the stainless steel is $T_{2,\text{mp}} = 1670\,\text{K}$. Additionally, the averaged thermophysical properties of the zirconia are taken from Schneider Jr [14]: ρ_1 $2300\,\text{kg/m}^3$, $c_1 = 750\,\text{J/kg.K}$, $k_1 = 2\,\text{W/m.K}$, $\alpha_1 = 0.13 \times 10^{-6}\,\text{m}^2/\text{s}$. The melting point of zirconia is $T_{1,\text{mp}} = 2953\,\text{K}$.

The important question that needs to be answered is the following: what is the maximum duration that is allowed for the transient ground test of the experimental rocket engine if the stainless steel must be maintained reasonably below its melting point of $T_{2,\text{mp}} = 1670\,\text{K}$?

Discussion of the computed temperature fields

First, to assess the quality of the NSM, a determination of the sensitivity of the two computed fields – the temperature field and the heat flux density field – to the cell population is an indispensable step. To comply with this, a series of numerical experiments was conducted which revealed at the end that the optimal number of cells needed is $N = 70$: The cells were unequally distributed in the following manner: $N_1 = 20$ in the zirconia and $N_2 = 50$ in the stainless steel. This choice ensures that the errors are nearly 1% at all compartments and guarantees that the temperature fields $T_1(x,t)$ and the heat flux density fields $j_1(x,t)$ in the zirconia ($0 < x < L_1$), together with the temperature fields $T_2(x,t)$ and the heat flux density fields $j_2(x,t)$ in the stainless steel ($L_1 < x < L_1 + L_2$) are accurately determined.

The first curve in Fig. 3 is $T_1(0,t)$ representing the temperature–time variation of the surface of the zirconia exposed to the turbulent stream of high-temperature combustion gases. It may be observed in Fig. 3 that this local temperature rises from the initial temperature, $T_i = 298$ K, to a very high temperature of 2275 K at the time of 744.5 s (roughly 13 min). Although this temperature is high, it is still far below the melting point of zirconia, $T_{1,\text{mp}} = 2953$ K (this temperature may be viewed as the secondary design constraint in the problem). Essentially, the shape of the first curve resembles a Heavyside step function.

The temperature–time variation of the zirconia/stainless steel interface, $T_{1,2} = T_1(L_1,t) = T_2(L_1,t)$, being the determining factor in this study, is plotted as the second curve in Fig. 3. Commencing with the initial temperature, $T_i = 298$ K, this critical temperature $T_{1,2}$ ascends in an exponential fashion and attains the melting point of the stainless steel, $T_{2,\text{mp}} = 1670$ K, at a time of 744.5 s (roughly 13 min). This melting point temperature constitutes the primary design constraint in the present unsteady heat conduction problem. Fundamentally, the elapsed time appended to $T_{2,\text{mp}}$ stipulates the maximum duration of the ground firing test of the experimental rocket engine. At this stage, it is worth mentioning that the pronounced separation between the first and the second curves at the beginning of the severe heating is a clear testimony to the massive conductive resistance imparted by the zirconia coating to the composite nozzle wall. This enormous conductive resistance creates a ‘thermal barrier’ to the heat transmitted from the stream of hot turbulent combustion gases to the stainless steel structure. As expected, the separation between the first and the second curves degrades with the passing of time. Quantitatively speaking, for an elapsed time of 744.5 s, the extent of the curve separation decreases to roughly one-third of its original size.

The third curve, indicative of $T_2(L_1 + L_2,t)$, in Fig. 3 bears lesser importance because it portrays the temperature change that the interface between the stainless steel and the insulation experiences as time progresses. Incidentally, it may also be noted that the shape of the third curve does not deviate markedly from that of the second curve. In fact, as time passes, the separation between the second and the third curves tends to diminish until it ultimately disappears for large time. As expected, the proximity of these two curves is a clear indication that the stainless steel does not offer a sizable conductive resistance (‘heat barrier’) to the heat transmission from the hot combustion gases to the composite nozzle wall. Essentially, at the crucial time of 744.5 s, the temperature of the insulation in contact with the stainless steel stays around 1600 K. Obviously, this large temperature has to be within the operational temperature range that the insulation can withhold, as

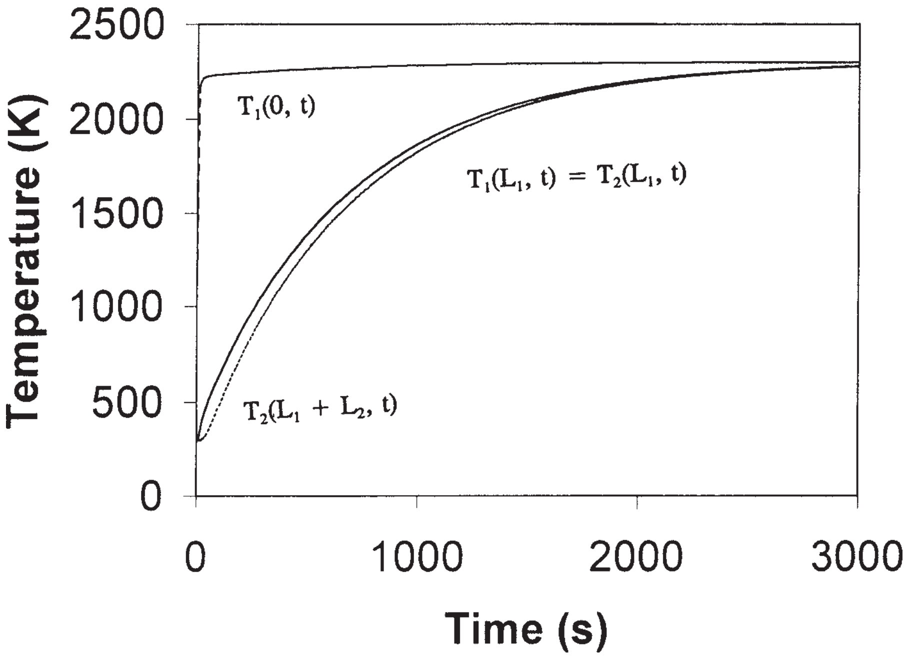

Fig. 3 *Temperature–time variation of: (a) the surface of zirconia exposed to the hot combustion gases, (b) the zirconia–stainless steel interface and (c) the stainless steel–insulation interface.*

suggested by the manufacturer [15]. Otherwise, the insulation disintegrates almost immediately.

Conclusions

The NSM, in combination with the code PSPICE, has proved to be an excellent tool for the concurrent prediction of the temperature field and the heat flux density field in layered plane walls, like the composite nozzle wall of an experimental rocket engine. The utility of these tools has been tested in classroom and laboratory environments devoted to heat transfer. Overall, the success of the duo NSM and PSPICE has been remarkable, demonstrating that the implementation of these tools are within the scope of undergraduate students on heat transfer courses.

The advantages that NSM offers over the finite-difference technique for the unsteady thermal analysis of composite walls have been discussed at length. The salient feature of the NSM is that the interface boundary conditions dealing with temperature continuity (equation 5), and with heat flux continuity (equation 6), in the composite wall are regulated automatically because of the satisfaction of the current conservation law in the electric circuits of the network.

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