A slider-crank experiment to determine the action of static forces

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Abstract A simple experiment to demonstrate the action of static forces in a slider-crank mechanism is described. Despite the apparent simplicity of the set-ups used, the experiment was able to produce measurements that were close to those predicted by kinetics analysis theory. More importantly, it was highly successful in helping students grasp the fundamental concepts of static forces operating in slider-crank mechanisms in an experiential manner.

Keywords slider crank; kinetics; theory of machines; engineering experiment

One of the most important and common mechanisms is the slider-crank. It is found in pumps, compressors, steam engines, feeders, crushers, punches and injectors. Furthermore, the slider-crank mechanism is central to diesel and gasoline internal combustion engines, which play an indispensable role in modern living.

The kinematics and kinetics of the slider-crank mechanism is well explained in many textbooks on the mechanics of machines [1, 2]. Recently, there has been some work reported on the use of specific software [3] and spreadsheets [2] to assist in creating diagrams for students to understand the kinematics of slider-crank mechanisms. However, a proper understanding of the kinetics is just as crucial as the kinematics of mechanisms. While a mechanism is designed primarily to provide motion, it is expected to bear loads throughout its service life. In terms of the kinetics aspect of mechanisms, physical experimentation is arguably more appropriate than software in conveying the essential concepts to students. The loads borne by the slider-crank mechanism may be static or dynamic. This paper details the development of an experiment to illustrate the action of static forces in a slider-crank mechanism.

Theory

Fig. 1 shows a slider-crank mechanism. Let the force acting on the slider be F_p, the velocity of the slider be v_p, the force acting perpendicularly to the crank be F_c, and the velocity of the crank at C be v_c. In the absence of friction, conservation of energy gives:

$$F_p v_p = F_c v_c \tag{1}$$

If PC is produced to intersect a vertical to O at M, replacing v_p and v_c, as described in equation (A3) of the Appendix, from equation (1), with OC and OM gives:

$$F_p OM = F_c OC = T \tag{2}$$

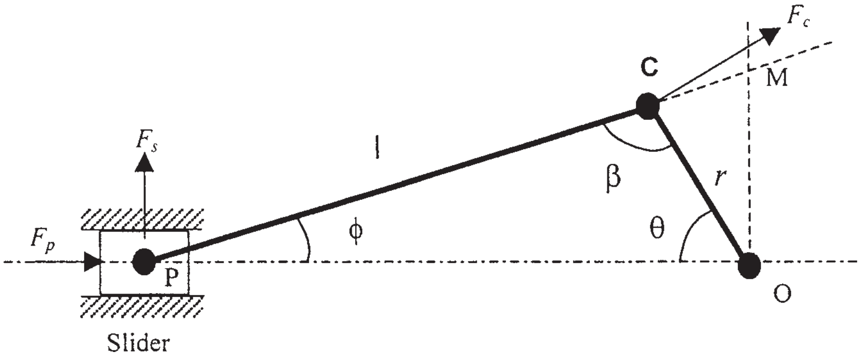

Fig. 1 *Schematic description of a slider-crank mechanism.*

where T is the torque developed at the crank. From the geometry of the system, we have:

$$r\sin\theta = l\sin\phi \tag{3}$$

Values of r, l and θ are typically known. Hence, this allows ϕ to be determined using:

$$\phi = \sin^{-1}\left(\frac{r\sin\theta}{l}\right) \tag{4}$$

The geometry of the system also provides the relation:

$$\tan\phi = \frac{OM}{r\cos\theta + l\cos\phi} \tag{5}$$

From equations (2) and (5), the slider force, F_p, required to maintain equilibrium with a crank torque, T, in the absence of friction is given by:

$$F_p = \frac{T}{\tan\phi(r\cos\theta + l\cos\phi)} \tag{6}$$

Under the action of force F_p on the piston, a side thrust, F_s, develops. It is expressed by:

$$F_s = F_p\tan\phi \tag{7}$$

Suppose that the slider is not frictionless, but has a coefficient of friction μ with the motion guides of the mechanism. The effective force, F_p', needed at the slider to maintain equilibrium with the crank torque, T, is now given by:

$$F_p' = F_p - \mu F_p\tan\phi \tag{8}$$

The coefficient of friction can be determined as shown in Fig. 2. Suppose the slider has mass m and is tilted at some angle, φ, to the horizontal. At the instant when the object first starts to slide,

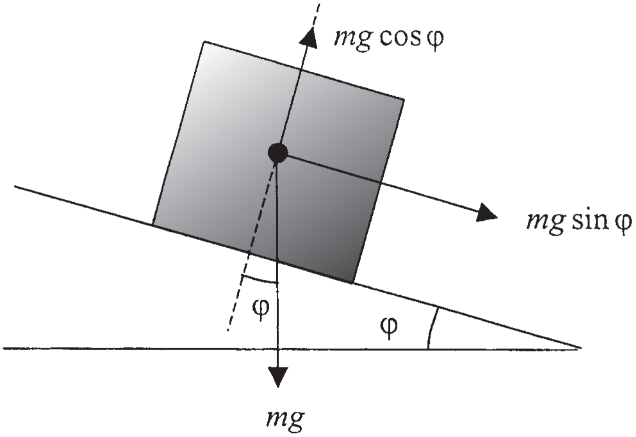

Fig. 2 *Free body diagram showing the forces acting on a mass on an inclined plane.*

$$mg\sin\varphi = \mu mg\cos\varphi \tag{9}$$

In other words,

$$\mu = \tan\varphi \tag{10}$$

Another important parameter in any linkage mechanism is the transmission angle. The transmission angle determines the effective transfer of torque from an input to output linkage. In the slider-crank mechanism, this can be determined by geometry using:

$$\beta = 180° - \phi - \theta \tag{11}$$

Experimental equipment and procedure

The central set-up used in the experiment was a slider-crank mechanism assembly that comprised (see Fig. 3) a base, circular crank, cord connected to the circular crank, connecting rod, slider, and force transducer attached to the slider. The other items used in the experiment were a vernier calliper, a fixture to anchor the force transducer, two C clamps, a plumb, six detachable weighting pieces of 0.25 kg mass each, and a strain gauge meter (P-3500 model by the Measurements Group). The force transducer was based on a strain gauge proving ring design, wherein the strain gauge used was a single element with 120 ohms resistance, 2.12 gauge factor, and 3 mm gauge length. The quarter-bridge configuration was applied for all readings made on the strain gauge meter.

The first part of the experiment involved determining the relevant physical parameters of the slider-crank mechanism. To do so, the connecting rod was detached, and the vernier calliper used to measure the connecting rod length *l*, crank radius *r*, and radius of the circular crank.

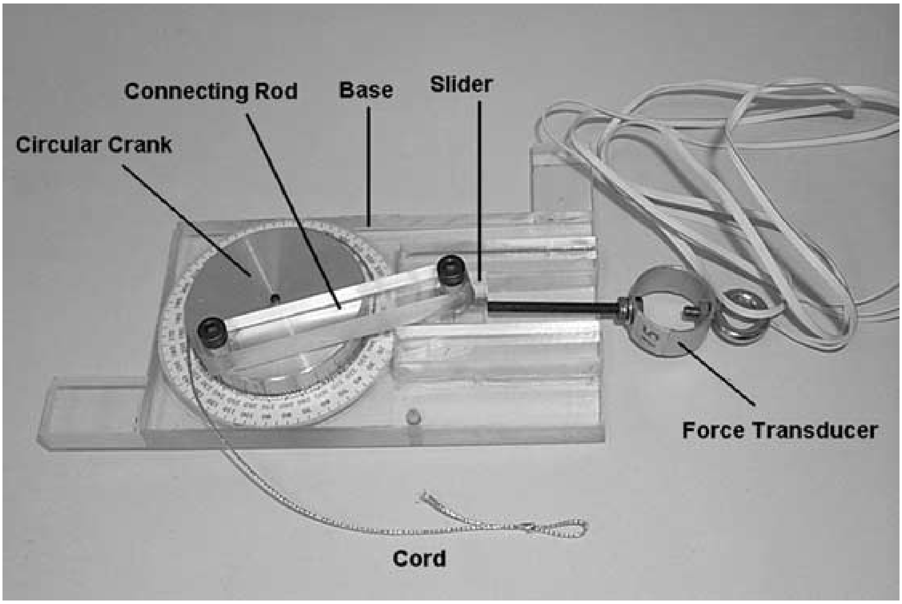

Fig. 3 *The experimental slider-crank mechanism assembly.*

The second part of the experiment involved determining the coefficient of friction between the slider and motion guides of the mechanism. The slider mechanism, with the connecting rod detached, was placed on the table. The plumb was then hung from the centre of the circular crank. As soon as the plumb was no longer swinging, the mechanism was slowly titled (see Fig. 4). At the instant the slider first started to move, the angle φ was recorded and μ calculated using equation (10).

The third part of the experiment involved calibrating the force transducer. For this part, the connecting rod was reconnected to the slider-crank mechanism and the mechanism firmly attached on the table using a clamp, in the manner shown in Fig. 5. The wires from the force transducer were attached to the strain gauge meter. The weights were hung from the force transducer, from 0 to 1.5 kg, at intervals of 0.25 kg. At each loading interval, the strain gauge reading was recorded.

In the last part of the experiment, the action of the static forces was investigated. The clamp was removed and reattached firmly on the slider-crank mechanism, as shown in Fig. 6. For the first reading, the crank angle was set at 40 °. The force transducer on the slider-crank mechanism was anchored to the fixture. The second clamp was used to ensure that the fixture was stationary. A 0.5 kg mass was then applied by hanging two pieces of the detachable weight at the end of the cord wrapped around the circular crank. This would produce a torque equal to the product of weight and radius of the circular crank. The strain gauge reading was then recorded. This process was repeated by increasing the crank angle at intervals of 10 ° to 130 °.

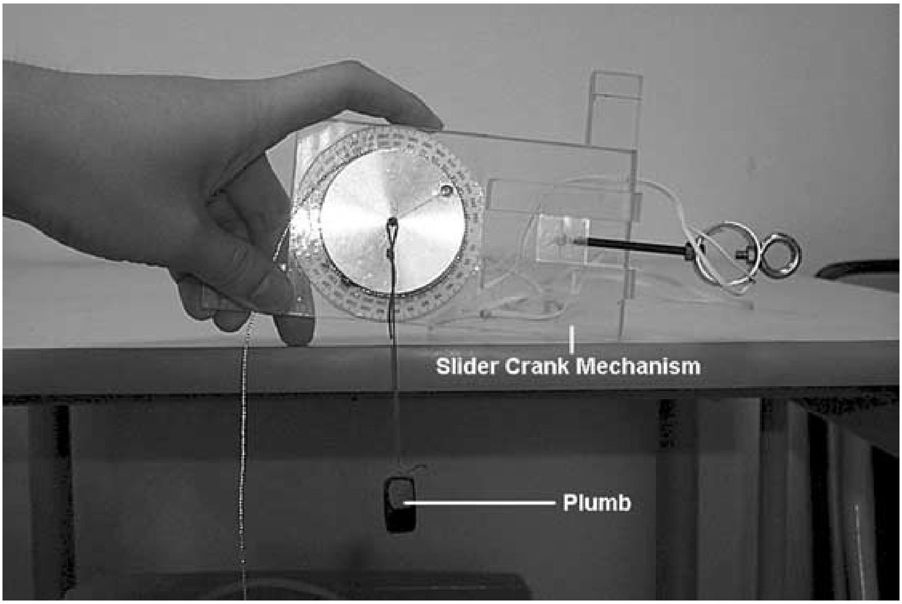

Fig. 4 *The experimental set-up used to determine the coefficient of friction.*

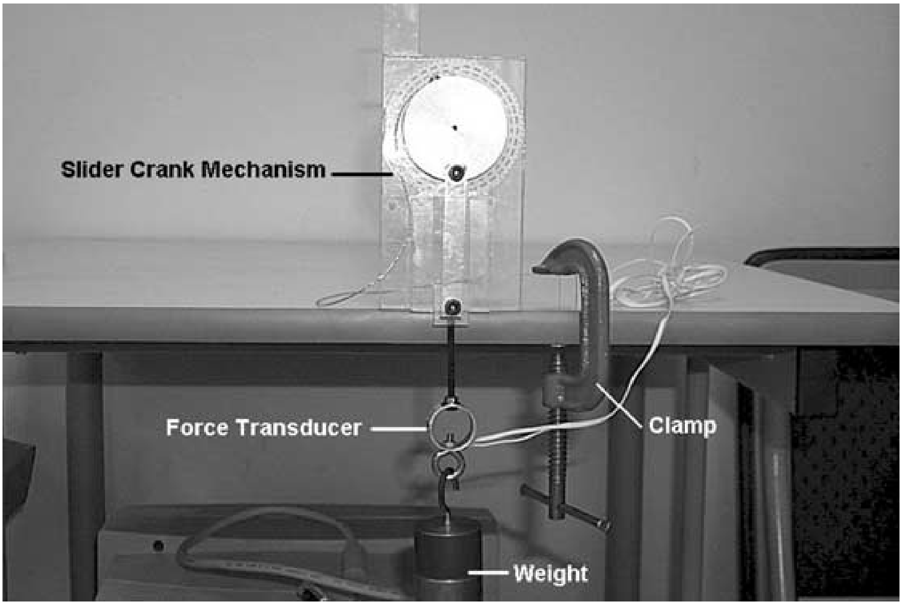

Fig. 5 *The experimental set-up used to calibrate the force transducer.*

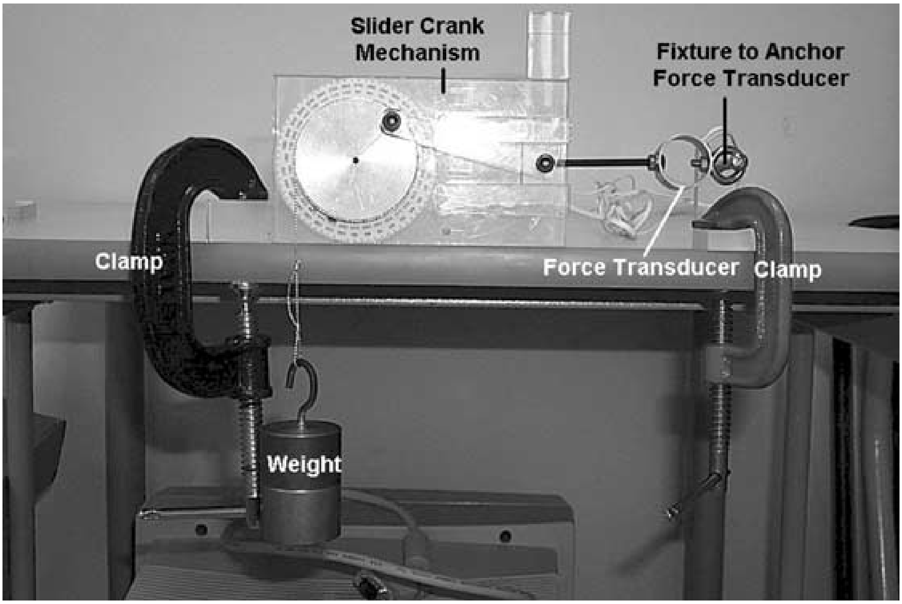

Fig. 6 *The experimental set-up used to determine the action of the static forces.*

Experimental results and discussion

As anticipated, students found the experiment easy to understand and conduct. They were encouraged to record their data in a spreadsheet program like Excel to facilitate the plotting of graphs and statistical analysis of results.

The following results were taken from a sample run of the experiment. From the first part of the experiment, the connecting rod, crank radius and radius of the circular crank were found to be 72 mm, 25 mm, and 27 mm, respectively. From the second part of the experiment, the slider first moved when the assembly was tilted at 45 °. This meant that the coefficient of friction was 1. This value was used in the theoretical calculations of F and F'.

Table 1 shows the results obtained from the third part of the experiment, to calibrate the strain-gauge force transducer. From these results, a graph of strain against force was plotted (see Fig. 7). The trend of the graph was noticeably linear. In fact, a computation of the R-squared value (used to determine the degree of linearity [4]) revealed a value of 0.9998, which was very close to the maximum limit of 1. This graph was then used to determine the force readings in the next part of the experiment.

Table 2 shows the results related to the fourth part of the experiment, which included values of F and F' calculated from theory and F' determined by experiment. From these results, a graph of these three parameters was plotted against the crank angle (see Fig. 8). It can be seen that the theoretical and experimental values

TABLE 1 *Values from the experiment to calibrate the force transducer*

Weight (N)	Strain reading ($\times 10^{-6}$)
2.45	50
4.91	95
7.36	148
9.81	190
12.27	238
14.72	290

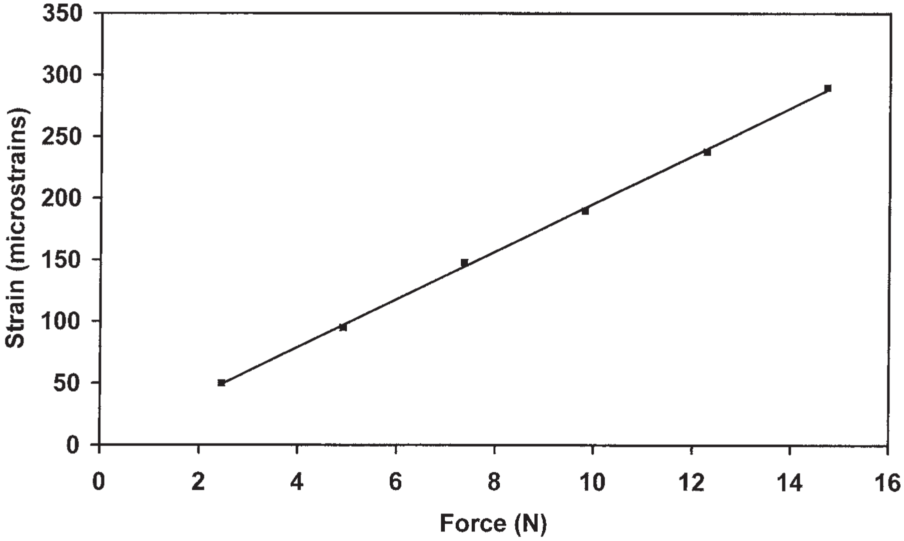

Fig. 7 *Graph of strain against force from the force transducer calibration experiment.*

of F' corresponded closely. This demonstrated the validity of the experiment. From the distributions of F and F', the effect of friction on static equilibrium is clearly demonstrated. One interesting question that can be posed about this comparison is the effect of starting crank torques in engines. Students can be guided to understand how friction leads to the reduction in effective driving force at the pistons. The last column in Table 2 gives the values of the transmission angle in relation to the crank angle in the experiment. Students should be guided to observe that the lowest force needed to maintain equilibrium occurred when the transmission angle was 90°. In many books on the mechanics of machines, this angle is often stated (sometimes without proof) as the most effective in the transmission of torque for linkage mechanisms.

TABLE 2 *Values from the experiment to determine the action of the static forces*

θ (degrees)	F: theory (N)	F': theory (N)	F': experiment (N)	β (degrees)
10	45.47	31.13	–	166.54
20	23.33	15.97	–	153.18
30	16.24	11.12	–	140.00
40	12.95	8.87	9.22	127.10
50	11.23	7.69	7.65	114.57
60	10.35	7.09	7.26	102.49
70	10.01	6.86	6.87	90.95
80	10.11	6.92	7.06	80.00
90	10.59	7.25	7.46	69.67
100	11.49	7.87	8.04	59.99
110	12.89	8.82	10.20	50.94
120	14.94	10.23	12.16	42.48
130	17.97	12.31	–	34.55
140	22.63	15.49	–	27.08
150	30.42	20.83	–	19.97
160	45.95	31.46	–	13.15

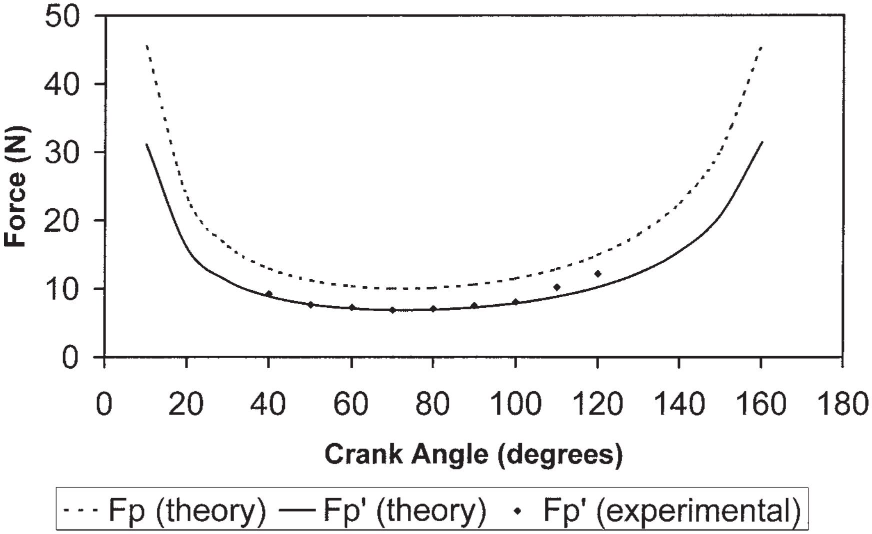

Fig. 8 *Graph of* F *(theory),* F′ *(theory), and* F′ *(experimental) against crank angle in the experiment to determine the action of static forces.*

Notwithstanding the closeness of the experimental and theoretical values in this sample experimental run, students should be encouraged to comment on possible causes of error. These may include inaccuracies from (a) friction at the mechanism joints, (b) incorrect setting of the crank angle, and (c) incorrect registration of strain due to poor clamping.

Some practical points should be noted with regard to the smooth running of this experiment. As weights were used, students should be warned to be careful in case they fall on some unsuspecting feet. Constant reminder is hence recommended. During the fourth part of the experiment, it was necessary to remove the mass and reapply the load a few times to ensure repeatable measurements. Readjustment of the anchor fixture may be needed if different readings are obtained each time. Students should also be reminded to keep within the range of crank angles prescribed. By intuition, angles close to 0° and 180° will result in very large values of F'. This will result in a sudden failure at some parts of the assembly. With these points in mind, careful experimentation should ensure that the set-ups are be re-usable over many times.

Conclusions

A cost-effective experiment to demonstrate the action of static forces in slider-crank mechanisms was found to yield results with good accuracy. The experiment was easy to conduct and contributed much towards the experiential learning of students.

References

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Appendix

Consider the graphical construction for velocity in a slider-crank mechanism shown in Fig. A1. Let ω be the angular velocity of crank OC and Ω the angular velocity of the connecting rod, PC. If I is the instantaneous centre for PC,

$$\Omega = \frac{v_{p/c}}{PC} = \frac{v_c}{IC} = \frac{v_p}{IP} \tag{A1}$$

Simple rearrangement gives:

$$v_p = v_c \frac{IP}{IC} \tag{A2}$$

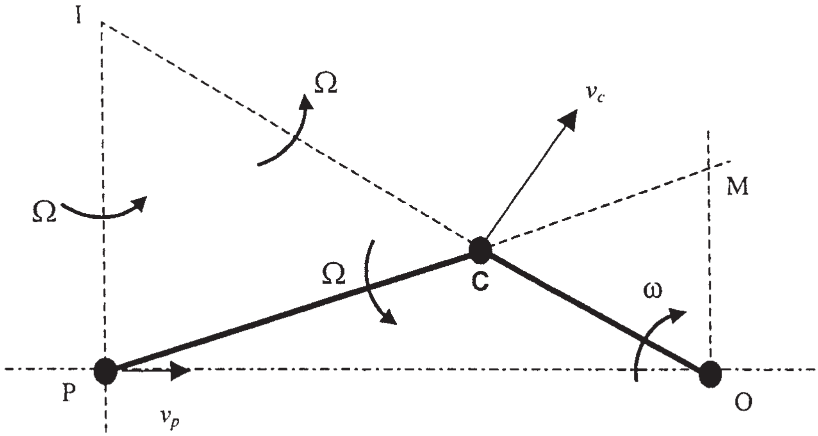

Fig. A1 *Graphical construction of velocity components in a slider-crank mechanism.*

If PC is produced to intersect a vertical to O at M, triangles PIC and OCM are similar. Hence, substituting IP and IC for OM and OC gives:

$$\frac{v_p}{v_c}=\frac{OM}{OC} \tag{A3}$$