The RC analogy provides a versatile computational tool for unsteady, unidirectional heat conduction in regular solid bodies cooled by adjoining fluids

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Abstract Evaluation of spatio-temporal temperatures and total heat transfer rates in simple bodies (large plate, long cylinder and sphere) has been traditionally explained in undergraduate courses of heat transfer by the Heisler/Gröber or by the Boelter/Gröber charts. These three charts pose some restrictions with respect to the applicable times. Additionally, the charts do not provide information about the time-dependent heat fluxes at the surface. Conversely, evaluation of spatio-temporal temperatures, time-dependent heat fluxes at the surface and total heat transfer rates can be easily done for the entire time domain with the network simulation method (NSM) in conjunction with the commercial code PSPICE. NSM relies on the existing physical analogy between the unsteady transport of electric current and the unsteady transport of unidirectional heat by conduction. This analogy has been named the RC analogy in the specialized literature. The code PSPICE simulates the electric circuits for a specific body together with the imposed boundary and initial conditions, and produces numerical results for the quantities of interest, such as: the spatio-temporal temperature distributions; the time-dependent heat flux distributions at the surface; and the total heat transfer.

Keywords unidirectional heat conduction; simple bodies; RC analogy; local temperatures; total heat transfer; PSPICE code

Notation

A	surface area of regular body
Bi	Biot number, $\bar{h}R/k_s$
c	geometric parameter: 0 for large plate, 1 for long cylinder and 2 for sphere
c_v	specific heat capacity
C_i	capacitance of cell i
E	voltage
$\bar{h}$	mean convection coefficient
j	heat flux density, electric current
k	thermal conductivity
m	mass
$\overline{Nu}$	mean Nusselt number, $\bar{h}(2R)/k_f$
N	number of cells
R_i	resistance of cell i
Q_{in}	initial heat stored
Q_{ins}	instantaneous heat transfer

Q_t total heat transfer
r transverse coordinate
R semi-thickness of large plate, or radius of long cylinder, or radius of sphere
t time
T temperature
T_c centre temperature
T_m mean temperature
T_w surface temperature
T_∞ free-stream temperature of fluid
V volume

α thermal diffusivity, $k/\rho c_v$
β coefficient of volumetric thermal expansion
ε total hemispherical emissivity
σ Stefan–Boltzmann constant
θ dimensionless T
η dimensionless r
ρ density
τ dimensionless t or Fourier number
Ω_t dimensionless Q_t

Subscripts
i initial
f fluid
s solid
w surface

Authors of textbooks on basic heat transfer cite the method of separation of variables as the primary procedure for the study of unsteady unidirectional heat conduction in regular bodies (large plate, long cylinder and sphere) that are exposed to gaseous or liquid convective environments (see for instance the textbooks by Incropera and DeWitt [1], Mills [2] and Thomas [3]). This elegant technique leads to infinite Fourier series and infinite Fourier–Bessel series which are capable of predicting the spatio-temporal temperature distributions in the body, the surface heat flux at the body surface, and the total heat transfer between the regular body and the surrounding fluid. For purposes of numerical evaluation of the local temperatures and the total heat transfer in real world situations, the singular characteristic of these infinite series is their rapid convergence for very long times. However, these infinite series diverge markedly for short times and many terms in the series have to be retained to secure adequate accuracy; the number of terms increases as time approaches zero.

In 1945, Boelter *et al.* [4] evaluated the three 'infinite series' (perhaps with few terms) and developed temperature–time charts for regular bodies (large plate, long cylinder and sphere) for the dimensionless time domain: $0 < \tau < 1.5$. The textbooks by Welty [5], Lienhard [6] and Suryanarayana [7] are the only ones on the US market that present the entire collection of the original Boelter charts for the three basic configurations. Reasonable simplifications in the 'one-term series' were exploited by Heisler [8] in 1947, enabling him to construct 'long-time' temperature–time charts for regular bodies (large plate, infinite cylinder and sphere). Unquestionably, over the years the 'long-time' Heisler charts have become a trademark of almost all textbooks on basic heat transfer in the modern era. Certainly, the precision exhibited by this kind of approximate solution based on the 'one-term series', when compared with the exact solutions based on the 'infinite series', depends on intrinsic errors, which in turn depend on time. Grigull *et al.* [9, 10] investigated the extent of these errors and came to the conclusion that they are within 1% when the dimensionless times, τ, exceeded 0.18 for the large plate, 0.21 for the long cylinder and 0.24 for the sphere. For conciseness, authors of textbooks that present the 'long-time' Heisler charts have averaged out these threshold numbers into an approximate figure-of-merit for a critical dimensionless time, equal to 0.2 [1–3]. It is worth mentioning that Heisler [8] also included in his seminal paper a companion set of 'short-time' temperature–time charts for the centre and the surface of each of the three body shapes. These supplementary charts are capable of handling the demanding dimensionless time sub-domain, $0 < \tau < 0.2$, which was uncovered in the 'long-time' version of the charts (applicable to the time sub-domain $\tau \geq 0.2$). Incidentally, it should be added as a side comment that the 'short-time' Heisler charts are never included in textbooks on basic heat transfer. Calculation of the total heat transfer rates has been normally explained in textbooks with the Gröber charts [11, 12]. which are sound for the time sub-domain $\tau \geq 0.2$.

On the other hand, it is widely known that all textbooks on heat transfer incorporate the electric circuit analogy, albeit on a limited basis [1–3, 5–7]. The three areas of application are: (a) steady conduction in walls that conform to rectangular, cylindrical and spherical coordinates, (b) unsteady conduction via a lumped formulation valid for Biot numbers < 0.1, and (c) steady radiation exchange between surfaces in enclosures. However, in connection to (b), solutions of the broader differential formulation for unsteady conduction of regular bodies (infinite plate, infinite cylinder and sphere) with Bi > 0.1 has never been attempted with the electric circuit analogy.

The central objective of this paper on engineering education is to provide students with an alternative, computer method that enables them to determine the temperature–time histories, the heat flux–time histories and the total heat transfer inside regular bodies for any period of time ($0 < \tau < \infty$), quickly, directly and accurately. The backbone of the paper focuses on the existing analogy between the unsteady transport of electric current and the unsteady transport of heat by conduction, the so-called RC analogy. As already mentioned in (a), this analogy is employed at length for steady conduction in walls of regular shape absent of heat generation. For unsteady heat conduction, the main highlight of this analogy stems from the fact that it uses discrete intervals of the space variable and real, continuous time for the independent variable.

Another potential use of the electric analogy lies in the analysis of unsteady conduction heat transfer under the influence of natural convection and/or radiation cooling. These two mechanisms are non-linear and owing to this complexity their

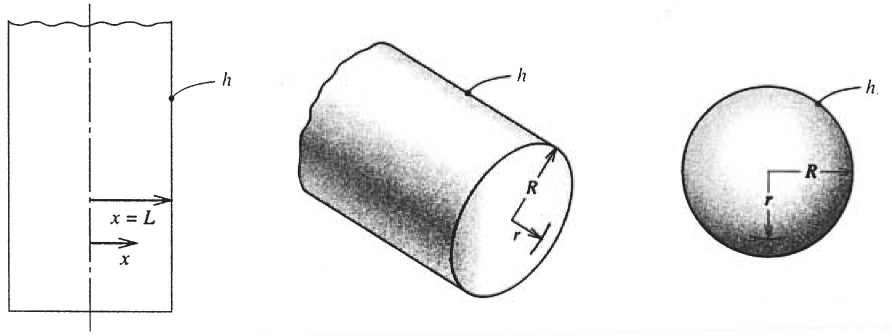

Fig. 1 *The three regular bodies.*

mathematical formulations are not compatible with the prevalent linearity that is the trademark of the group of Boelter, Heisler and Gröber charts.

Mathematical formulation

Cooling by forced convection

Consider unsteady 1-*D* heat conduction in a solid body (large plate, long cylinder and sphere) possessing a uniform temperature T_i. For $t > 0$, the body is immersed in a fluid and the surface of the body commences to exchange heat by forced convection with the fluid. A constant free-stream temperature, T_∞, and a mean convective coefficient, $\bar{h}$, characterize the fluid. The three bodies under study here and the corresponding coordinate systems are illustrated in Fig. 1. According to this general description, the mathematical formulation for solid materials with constant specific heat capacity, c_v, and constant thermal conductivity, k, is given by the linear heat conduction equation:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{c}{r}\frac{\partial T}{\partial r} \tag{1}$$

The initial and boundary conditions are given by:

$$T = T_i, \quad t = 0 \tag{2}$$

$$j = k\frac{\partial T}{\partial r} = 0, \quad r = 0 \tag{3a}$$

$$j = -k\frac{\partial T}{\partial r} = \bar{h}(T - T_\infty), \quad r = R \tag{3b}$$

In the second term of the right-hand side of equation (1), c stands for a geometric parameter taking values of 0 for a large plate, 1 for a long cylinder and 2 for a sphere.

From the standpoint of conduction heat transfer, the dimensionless form of the mean convection coefficient, $\bar{h}$, appearing in equation (3b) is identified as the Biot number, Bi = $\bar{h}R/k_s$. However, quantitative information about $\bar{h}$ usually originates in forced heat convection of fluids [1–3, 5–7] in the form of theoretical or empirical correlation equations for the mean external Nusselt number, $\overline{\text{Nu}} = \bar{h}(2R)/k_f$ (another dimensionless form of the mean convection coefficient, $\bar{h}$). Therefore, the connecting bridge between Bi and $\overline{\text{Nu}}$ turns out to be:

$$\text{Bi} = \frac{1}{2}(k_f/k_s)\overline{\text{Nu}} \tag{4}$$

where the proportionality constant is the ratio of the thermal conductivity of the fluid, k_f, and the thermal conductivity of the solid, k_s.

Once the temperature distribution $T(r,t)$ has been determined, the mean temperature, T_m, is computed from the integral

$$T_m(t) = \frac{1}{V}\int_V T(r,t)dV \tag{5}$$

where V is the volume of the body.

The instantaneous heat transfer, Q_{ins}, at the body surface may be obtained mathematically in two ways: one is utilizing Fourier's law and the other is employing Newton's 'law' of cooling. The latter is preferred here, resulting in:

$$Q_{ins}(t) = jA = -\bar{h}A[T(R,t) - T_\infty] \tag{6}$$

Thereafter, the total heat transfer, Q_t, between the body and the fluid in an elapsed period of time from 0 to t_f is determined from the integral

$$Q_t = \int_0^t Q_{ins}(t)dt \tag{7}$$

In other words, Q_t represents the total amount of energy that is lost (or gained) by the body in the chosen time interval. Setting aside the indirect mathematical approach, Q_t may be rapidly obtained with a direct physical approach by way of a global thermodynamic energy balance:

$$Q_t = mc_v[T_i - T_m(t)] \tag{8}$$

where T_m is given by equation (5).

Cooling by natural convection

The mean convection coefficient, $\bar{h}$, that governs natural convection cooling from the surface of a body to a fluid is not a constant quantity, as happens in forced convection cooling. Essentially, $\bar{h}$ is susceptible to a function of the temperature difference, $(T - T_\infty)^n$ [1–3, 5–7], where T is the surface temperature of the body, T_∞ is the temperature of the fluid, and the exponent n absorbs values of $\frac{1}{4}$ and $\frac{1}{3}$ for laminar and turbulent flows, respectively. In view of the non-linear behaviour of $\bar{h}$, it is expected that the strategy for the analysis of unsteady heat conduction with natural convective cooling deviates slightly from the one for forced convection cooling and should be more elaborate.

In addition, a common belief in heat transfer calculations is that whenever the fluid (the thermodynamic sink) is air, and if the initial-to-air temperature difference ($T_i - T_\infty$) is not excessively high, natural convection eclipses surface radiation. However, since for natural convection heat transfer the magnitudes of $\bar{h}$ are relatively low, it is presumed that radiation heat transfer may play an important role, particularly if the emitting surface of the body is nearly black, to deliver a high value for the total hemispherical emissivity, i.e., $\varepsilon \to 1$. In order to learn whether natural convection dominates, radiation dominates or even natural convection and radiation coexist, Mills [2] has recommended checking the contribution of each mechanism separately. Certainly, for steady cooling this calculation is purely arithmetical, but for unsteady cooling the calculation requires the solution of a partial differential equation (1), whose dominant boundary condition at the surface is affected by the addition of two heat fluxes (one by natural convection and the other by radiation). That is, the actual non-linear boundary condition is written as:

$$j = -k\frac{\partial T}{\partial r} = \bar{h}(T - T_\infty) + \varepsilon\sigma(T^4 - T_\infty^4), \quad r = R \tag{9}$$

where $\bar{h} = C(T - T_\infty)^n$ and C is a constant. Thereby, under these broader circumstances, the establishment of definite borderlines between the two competing heat transfer mechanisms is not an easy task.

The RC analogy and the network simulation method

The foundation of the RC analogy reposes on the physical analogy between the unsteady theory of electric circuits and the unsteady theory of heat conduction. The exploitation of this analogy has its genesis in the pioneering work of Paschkis and Baker [13], who successfully solved the unidirectional linear heat conduction equation for a large plate using an analogue computer. This massive apparatus, named a heat and mass flow analyser, was in reality a tailor-made electric network composed of resistors and capacitors whose interconnectivity facilitated the duplication of real heat conduction problems in solid bodies. These authors measured the instantaneous voltage differences and currents in the analyser, which, in the spirit of the analogy, corresponded to temperature differences and instantaneous heat transfer, respectively.

For the sake of simplicity, the general procedure for implementing the NSM has been delineated for a large plate of semi-thickness L in rectangular coordinates, but extensions to long cylinders and spheres in their appropriate coordinates is a straightforward procedure. Essentially, the NSM consists in dividing the physical domain $0 < x < L$ into a finite number of cells of equal thickness, Δx, creating sort of a computational domain. An energy balance in a typical cell, $i = 1, 2, 3, \ldots, N$, may be expressed as:

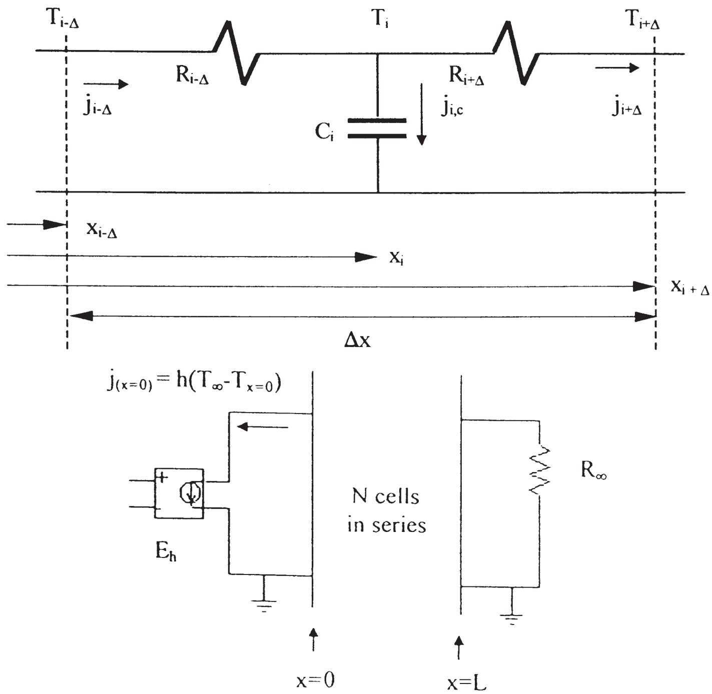

Fig. 2 *Electric network model for: (a) an elementary interior cell and (b) a cell for a convective surface.*

$$(\rho c)_i \frac{dT_i}{dt} = -\frac{\Delta j}{\Delta x} = \frac{j_{i-\Delta} - j_{i+\Delta}}{\Delta x} \tag{10}$$

where $j_{i-\Delta}$ and $j_{i+\Delta}$ signify the respective heat flux densities entering and leaving cell i, as indicated in Fig. 2a. In turn, these two heat flux densities are defined by Fourier's law as:

$$j_{i\pm\Delta} = \pm k \frac{T_i - T_{i\pm\Delta}}{\Delta x/2} \tag{11}$$

In this equation, $T_{i-\Delta}$, $T_{i+\Delta}$ and T_i denote the respective temperatures at the left extreme, at the right extreme and at the centre of cell i. Hence, the combination of equations (10) and (11) yields a generic differential–difference equation:

$$\Delta x(\rho c)_i \frac{dT_i}{dt} = k\frac{T_{i-\Delta} - T_i}{\Delta x/2} - k\frac{T_i - T_{i+\Delta}}{\Delta x/2} \tag{12}$$

for a typical cell $i = 1, 2, 3, \ldots, N$. Inspection of this equation reveals that the NSM is a physical-intensive numerical simulation, whose main feature is the use of discrete intervals of the space variable and real, continuous time as the independent variable.

At this point, each term of equation (12) is conveniently redefined by way of an electric current. This operation leads to:

$$j_{i-\Delta} - j_{i+\Delta} - j_{i,c} = 0 \tag{13}$$

where

$$j_{i,c} = \Delta x(\rho c)_i \frac{\mathrm{d}T_i}{\mathrm{d}t} \tag{14}$$

Both fields, the temperature field $T(r,t)$ and the heat flux density field $j(r,t)$ in the body in question, can be easily determined by the NSM.

In the context of the prevalent equivalence, equation (12) may be interpreted as Kirchoff's current law (implying energy conservation), where the temperature, T, is a continuous single-valued dependent variable that satisfies Kirchoff's voltage law. Under this umbrella, the stage is now set for analysing the time-dependent conduction of heat described by two characteristic variables, namely the temperature, T_i, and the heat flux density, j_i. Use is made of an equivalent electric network whose characteristic variables are the voltage, V_i, and the electric current, I_i. Within this analogue framework, equation (12) envisions the participation of two equal resistors with resistances

$$R_{i\pm\Delta} = \frac{\Delta x}{2k} \tag{15}$$

and one capacitor of capacitance

$$C_i = \Delta x(\rho c)_i \tag{16}$$

Accordingly, the interconnection between the two resistors $R_{i-\Delta}$ and $R_{i+\Delta}$ and the capacitor C_i in the sequence of cells $i - 1$, i and $i + 1$ is illustrated in the electric circuit of Fig. 2a. The generalization of these elements produces N cells which are connected in series to form the complete network model.

Finally, the last steps deal with the inclusion of the boundary and the initial conditions, equations (2) and (3), which needs special treatment. These steps are performed with specific electric devices in the following manner. A resistor of infinite value, R_∞ (an open circuit), is connected to the first cell, $i = 0$, to cope with the symmetrical boundary condition, equation (3a). A voltage-control current-source, G, is connected at the last cell, $i = N$, to handle the dominant convective boundary condition, equation (3b). The portions of the electric circuits that represent both boundary conditions are shown in Fig. 2b. At the end, the initial condition is adjusted by charging the capacitors of the network to the magnitude specified by the initial temperature in equation (2).

In this contemporary era, the computations of temperatures and heat flux densities no longer need to be performed with electric networks like the heat and mass flow analyser. With the advent of digital computers, the time-consuming operations executed by the researchers using such apparatus can be simulated numerically quickly and efficiently with a commercial code, PSPICE [14], for personal computers. Electrical engineers at the University of California at Berkeley designed a family of codes, SPICE [15], with the purpose of simulating the unsteady behaviour of complex circuits containing a variety of circuit elements, such as resistors, capacitors, voltage sources, current sources, etc. The functionality and adaptability of PSPICE has been tested thoroughly by Alhama [16] for the analysis of linear and non-linear problems of heat conduction in solid media.

Presentation of results

For convenience, let us adopt the customary set of dimensionless variables

$$\theta = \frac{T - T_\infty}{T_i - T_\infty}, \quad \tau = \frac{t}{R^2/\alpha_s}, \quad \eta = \frac{r}{R} \tag{17}$$

where the respective scales are: $(T_i - T_\infty)$, R^2/α_s (identified as the heat diffusion time in [17]) and R. As a consequence of the non-dimensionalization of equations (1)–(3), a dimensionless parameter, the Biot number, $\text{Bi} = \bar{h}R/k_s$, appears in the modelling. To facilitate the numerical computations, we assigned values of one to $T_i = R = \alpha_s$ and the value of zero to T_∞, leaving at the end Bi proportional to the ratio $\bar{h}/k_s$.

From geometric considerations, it is widely known that among the three basic shapes (large plate, long cylinder and sphere), the sphere exhibits the highest rate of heat transfer and, because of this peculiarity, we chose the sphere to conduct a comparison test.

A sensitivity analysis of the cell population in the computational domain of a sphere was carried out for the critical case of an isothermal surface characterized by a large Bi (= 100). A series of numerical experiments reflects that the optimal number of cells needed is about 100 to guarantee acceptable convergence for short times. This choice ensures that, irrespective of time, the intrinsic temperature errors and heat flux density errors are nearly zero for all Bi.

Cooling by forced convection

For purposes of comparing the numerical temperature results produced by the NSM, the baseline solutions may be chosen from two sources: (a) evaluation of the infinite series [10]

$$\theta(\eta, \tau) = 2\sum_{n=0}^{\infty} (-1)^{n+1} e^{-(\lambda_n R)^2 \tau} \frac{\sin[(\lambda_n R)\eta]}{(\lambda_n R)\eta} \tag{18}$$

where the eigenvalues are $\lambda_n R = n\pi$, $n = 1, 2, 3, \ldots$ or (b) reading the temperature–time pairs from the Boelter diagrams [4, 5–7]. We opted for the latter to expedite the comparisons.

An inspection of the Boelter diagrams for the sphere, reproduced in Fig. 3 (centre), and Fig. 4 (surface), can be conceived as follows. The initial temperature, $\theta = 1$, is

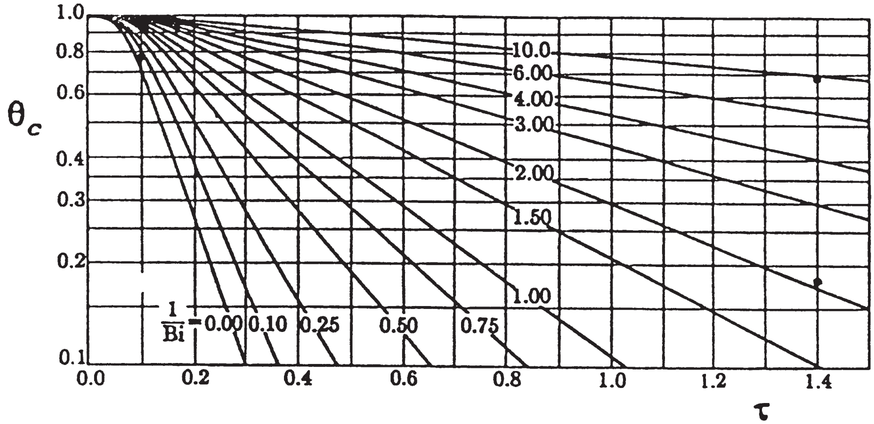

Fig. 3 *Centre temperatures in a sphere, obtained by the NMS, and those extracted from the Boelter diagrams [4–7].*

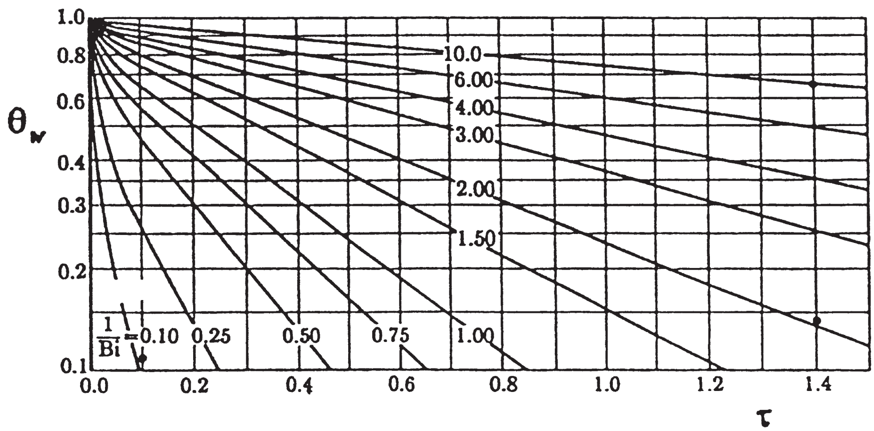

Fig. 4 *Surface temperatures in a sphere, obtained by the NMS and those extracted from the Boelter diagrams [4–7].*

located in the upper left corner of the two diagrams. For vigorous cooling, where the internal conductive resistance dominates the external convective resistance (implying large Bi), the final surface temperature, θ_w, occurs near the lower left corner of the diagram in Fig. 4. In fact, for Bi = 10, the variation of the surface temperature, θ_w, is very pronounced, dropping significantly from 1 to 0.1 in a very short period of time, of $\tau = 0.1$. This situation involving rapid cooling seems to be a crucial test for the NSM. The readings in the diagrams in Figs. 3 and 4 indicate that the exact centre temperature, θ_c, is 0.80 and the exact surface temperature, θ_w, is 0.10.

Our calculations with the NSM also give $\theta_c = 0.80$ and $\theta_w = 0.10$, in perfect agreement with the values from the Boelter diagrams.

For purposes of visualization, the centre and the surface temperatures computed by the NSM are indicated in Figs. 3 and 4 with black dots. Based on the excellent comparisons in the sphere, it may be inferred without hesitation that the large plate and the long cylinder will exhibit adequate levels of accuracy.

Calculation of the dimensionless total heat transfer, $\Omega_t = Q_t/Q_\infty$, between the initial time, $t = 0$, and a final time, t, in the three bodies may be computed directly and rapidly using the complement of the dimensionless mean temperature, θ_m:

$$\Omega_t = 1 - \theta_m(\tau) \tag{19}$$

where

$$\theta_m(\tau) = (c+1)\int_0^1 \theta \eta^c \, d\eta \tag{20}$$

The amount of heat stored initially in a body, Q_∞, is quantified by the limiting relation

$$Q_\infty = m_s c_s (T_i - T_\infty) \tag{21}$$

The thermodynamic procedure for Q_t pursued in this subsection seeks to supplant the use of the Gröber charts [11, 12] for the determination of the total heat transfer in the body.

Conclusions

This paper presented the NSM for the first time as a viable tool to teach unsteady heat conduction in regular solid bodies that will appeal to the inductive learner. An inductive learner prefers to proceed from specific to general, while a deductive learner prefers to proceed from general to specific. Studies have shown that induction promotes deeper learning and results in longer retention of the information by students. All textbooks on heat transfer [1–3, 5–7] present the RC analogy for the steady heat conduction in walls without heat sources. This information sets the ground for an inductive approach to teaching, where the next level revolves around the time-dependent RC analogy for solutions of unsteady heat conduction of regular bodies.

The versatility of the NSM coupled with the PSPICE code has been manifested in the presentation of results. This combination permits the rapid determination of the spatio-temporal variation of temperatures, the time-dependent heat flux distributions at the surface, the mean temperatures and the total heat transfer rates in regular bodies (large plates, long cylinders and spheres) for the entire dimensionless time domain ($0 < \tau < \infty$).

It should be emphasized that the NSM is capable of handling generalized boundary conditions (linear or non-linear) for asymmetric cooling of large plates, long hollow cylinders and spherical shells with a minor interchangeability of resistors and capacitors in the network.

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